Evaluation of Effective Astrophysical S factor for Non-Resonant Reactions

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We derived analytic formulas of the effective astrophysical S factor, $S_{\rm eff}$, for a non-resonant reaction of charged particles using a Taylor expansion of the astrophysical S factor, $S(E)$, and a uniform approximation. The formulas will be able to generate more accurate approximations to $S_{\rm eff}$ than previous ones.

Evaluation of thermonuclear reaction rates is very important for nucleosynthesis and energy generation in stars. The reaction rate at a temperature T is expressed in terms of an effective astrophysical S factor defined as [1)]

$$S_{\rm eff} = \sqrt{\frac{\tau}{4\pi}}\frac{e^{\tau}}{E_0}\int_0^{\infty} dE\, S(E) \exp\left(-\frac{E}{kT} - 2\pi\eta(E)\right) , \qquad \tau = \frac{3E_0}{kT} , \qquad (0\cdot1)$$

where k is Boltzmann's constant, E_0 the Gamow peak energy, $\eta(E)$ the Sommerfeld parameter, and $S(E)$ the astrophysical S factor [2)]. In many calculations of the stellar evolution approximations to $S_{\rm eff}$ are used. For instance, the approximate expression of $S_{\rm eff}$ obtained by the stationary phase approximation [2)] is made use of for non-resonant reactions of charged particles. Since the integrand in Eq. (0·1) is strongly peaked around E_0 (the Gamow peak) so that $S_{\rm eff}$ can be almost determined by the contribution from the integrand within an energy window of effective width $\Delta = 4\sqrt{E_0 kT/3}$ [2)]. On the other hand, demand for methods of evaluating $S_{\rm eff}$ more accurately increases as theories of stellar evolution are developed. Therefore, by using the uniform approximation [3)] and Taylor expansion of $S(E)$ around the threashold and the Gamow peak energy, we made the following two analytic formulas [4)] which can generate more accurate approximations to $S_{\rm eff}$

$$S_{\rm eff-thd} = \sum_{n=0}^{n_M}\frac{1}{n!}S^{(n)}(0)E_0^n\sum_{k=0}^{k_M}\frac{P_{2k}(n)}{k!(12\tau)^k} , \qquad S^{(n)}(0) = \left.\frac{d^nS}{dE^n}\right|_{E=0} \qquad (0\cdot2)$$

$$S_{\rm eff-Gam} = \sum_{n=0}^{n_M}\frac{1}{n!}S^{(n)}(E_0)E_0^n\sum_{r=0}^{n}(-)^r\left(\begin{array}{c} n \\ r \end{array}\right)\sum_{k=0}^{k_M}\frac{P_{2k}(n-r)}{k!(12\tau)^k} , \qquad (0\cdot3)$$

where $P_{2k}(n)$ is the polynomials of n as follwos:

$$P_0(n) = 1 , \qquad P_2(n) = 12n^2 + 18n + 5 ,$$
$$P_4(n) = 144n^2 + 336n^3 + 84n^2 - 144n - 35 ,$$

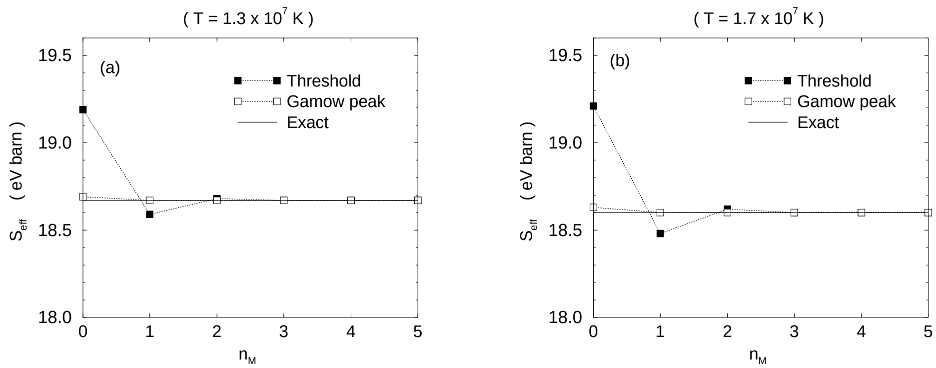

Fig. 1. Approximate values of $S_{\rm eff}$ generated by $S_{\rm eff-thd}$ (closed squares) and $S_{\rm eff-Gam}$ (open squares) together with the exact value of $S_{\rm eff}$ (the solid line) at (a) $T = 1.3 \times 10^7$ K and (b) $T = 1.7 \times 10^7$ K. The astrophysical S factor was Taylor-expanded up to n_M-th order.

$$P_6(n) = 1728n^6 + 4320n^5 - 4320n^4 - 13320n^3 - 288n^2 + 6210n + 665.$$

The values of n_M and k_M correspond to the numbers of terms in the Taylor expansion of $S(E)$ and the asymptotic expansion of $S_{\rm eff}$ (expansion parameter $1/\tau$), respectively. The analytic formulas are expected to be useful for high temperature enviroments and reactions which have strong E-dependence around the corresponding Gamow peak energies. Note that the formulas are valid if $E_0 + \Delta/2$ is within the radius of convergence concerning the Taylor expansion of $S(E)$.

We applied our formulas to the ^{7}Be$(p,\gamma)^8$B reaction in the stellar interior of the sun, which is reported to have strong E-dependence around the corresponding Gamow peak energy $E_0 \approx 20$ keV [5]. Here we employed the function form of $S(E)$ obtained in Ref. 5). Figs. 1-(a) and 1-(b) show the approximate values of $S_{\rm eff}$ for the reaction at $T = 1.3 \times 10^7$ K and $T = 1.7 \times 10^7$ K, respectively. In each figure closed and open squares represent the approximate values of $S_{\rm eff}$ generated by Eqs. (0·2) and (0·3), respectively. The exact values of $S_{\rm eff}$ is denoted by the solid line. Note that the two approximate values for a given n_M are not modified by inclusion of terms of higher order than $k_M > 1$. Therefore, only the approximate values with $k_M = 1$ are shown in the figures. At both temperatures $S_{\rm eff-thd}$ converges to the exact result with $n_M = 4$ and $k_M = 1$ while $S_{\rm eff-Gam}$ does with $n_M = 2$ and $k_M = 1$. The result can be understood by considering that $S_{\rm eff}$ is evaluated even quantitatively by $S(E)$ in the energy region $E_0 - \Delta \leq E \leq E_0 + \Delta$. However, it is necessary to confirm more systematically whether the uniform approximation works well in approximating $S_{\rm eff}$.

References

[1] J. N. Bahcall, *Neutrino Astrophysics* (Cambridge University, Cambridge, 1989)
[2] D. D. Clayton, *Principles of Stellar Evolution and Nucleosynthesis* (Chicago Press, Chicago, 1983)
[3] D. M. Brink, *Semiclassical Method in Nucleus-Nucleus Scattering* (Cambridge University, Cambridge, 1985)
[4] M. Ueda, A. J. Sargeant, M. P. Pato, and M. S. Hussein, Phys. Rev. **C61** (2000) 045801
[5] B. K. Jennings, S. Karataglids, and T. D. Shoppa, Phys. Rev. **C58** (1998) 3711