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Approximate analytical formulas for Kirchhoff migration operator

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Abstract

Two important tools in seismic processing—Kirchhoff migration and demigration operators—are the basis for many imaging problems solution. Due to high numerical and computational requirements, the use of those tools for three dimensions are very computation-costly. This fact has motivated us to investigate Kirchhoff migration operations for simpler types of media in order to provide faster results to be used as an approximation for more realistic media. To obtain results with lower computational effort, a convenient environment is the so called 2.5D situation, i.e., considering 3-D wave propagation in a medium that does not vary in the horizontal direction perpendicular to the seismic line. In this case, 2-D ray-tracing is sufficient to describe the 3-D propagation effects, particularly geometric-spreading. In a medium where the parameters depend only on the depth component (1-D situation), the imaging operations only require the solution of semi-analytical integrals, which can be both precisely and immediately implemented. For some particular cases of vertical velocity distributions, approximate analytical formulas are devised for migration stacking-lines and weight functions. Several imaging algorithms present very efficient computational performance by using those models. Thus, it is possible to establish a set of cases which may be useful for validating the implementation of more complex situations.

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1. Introduction

This work describes a class of results for stacking-lines and weight functions for some vertical velocity models. This class of results is indeed a set of terms of integral operators known in Geophysics as Kirchhoff-type integrals. Included in these operators are the so called Kirchhoff migration operators, which have applications on several subsurface image construction problems through seismic data processing. Multidisciplinary staff involving geophysicists, geologists and reservoirs engineers use seismic data acquired both on earth or sea surface for hydrocarbon reservoirs prospection and monitoring. In order to obtain these data, mechanical waves are generated and propagated towards the inner earth, being reflected back by discontinuity interfaces of geological parameters. The soil vibration movement (water, pressure) resulting from those reflected waves is recorded as a function of time by a geophone system (hydrophones). By this process, the Geophysics role is to rebuild a subsoil image from seismic data. For this purpose, a highly developed set of seismic processing methods are used (see [10]). One of the processes (or steps for the seismic processing sequence) to be used is the so called migration. The migration aims to transform (migrate) seismic data into image regions for the subsurface of interest using a subsoil seismic velocity initial model (macro-model), which is built by previous processes. Therefore, the migration is one of the most important seismic imaging operations. For several imaging processes, integral operators are used. In geophysical literature, it is usual to attribute Kirchhoff's name to those integrals. The reason for this is the fact that the integrals are linked with the so called Kirchhoff integral, which describes the wave propagation by models. Schneider in [9] established the integral operator for Kirchhoff migration. Later it was observed that this migration scheme is equivalent to the ''stacking diffraction'' earlier proposed by Rockwell in [7] using the Hagedoorn Maximal convexity surfaces, which are currently known as diffraction surfaces or Huygens surfaces. Thus, both Rockwell in [7] and Schneider in [9] works state that stacking data along the Huygens surfaces and placing the obtained results for each corresponding in-depth point make it possible to produce an in-depth image of the subsurface, if a velocity model and a source receptor are known. Bleistein in [2] and Schleicher in [8] present weight functions for the migration operation, aiming to provide an image (migrated section) on which amplitudes have been compensated by the geometrical-spreading factors. The waves can be calculated using ray tracing (dynamically) in the velocity model (see [3]). Due to high computational cost for Kirchhoff-type methods, simplified representations are used in practice (see [4]). Within this context and aware of discussions previously published on restricted medium, such as the 2.5D and vertically inhomogeneous medium (see [1]), new perspectives for further investigations became evident. In other words, problems for those media would have better chances to achieve similar performance in less time and at a lower computational cost due to their special characteristics. For those media, the terms appearing in the integrands of the Kirchhoff operators have analytical formulas which are of prompt implementation. The purpose of this work is to establish the migration stacking curves and weight functions for Kirchhoff-type migration for some vertical velocity distribution models (constant gradient slowness, constant gradient of quadratic velocity and constant gradient of cubic velocity).

2. Migration integral

This section presents the migration integral and the expressions pertaining to weight functions and migration stacking curves for a vertically inhomogeneous medium. Initially, the seismic register is supposed to be composed of analytical traces (analytical means it is formed from the real register, source signal, having the Hilbert transform as the imaginary part). These traces are superpositions of events of primary reflections specified by $U(\xi,t) = U(\mathbf{s}(\xi),\mathbf{g}(\xi),t)$ and usually well described by zero-order ray theory, i.e., $U(\xi,t) = U_0(\xi)F(t-\tau_\mathrm{R}(\xi))$ where τ_R is the transit time along the reflection of the source–receptor pair in the specified seismic configuration through parameter ξ, localization, $U_0(\xi)$ is the amplitude factor and $F(t)$ is the analytical signal.

This representation is performed through a large sum of objects defined in a set called the aperture set (domain constituted by the corresponding source-and-receiver parameters). These objects are decomposed in a two-factor product; the first is a weight function, the second elementary seismic reflections, where inserted amplitudes are distributed in a seismic section along the diffraction surface (see [8]).

The 2.5D migration integral is given by (see [5]),

$$V(\mathbf{m}) \simeq \frac{1}{\sqrt{2\pi}} \int_{a_1}^{a_2} \mathrm{d}\xi W_{DS}^{(2.5\mathrm{D})}(\xi,\mathbf{m}) \frac{\mathrm{d}^{1/2}}{\mathrm{d}(-t)^{1/2}} U(\xi,t)\bigg|_{t=\tau_D(\xi,\mathbf{m})}, \tag{1}$$

where $\mathbf{m} = (x,z)$ denotes the in-depth fixed point where the migration $V(\mathbf{m})$ shall be described, $\frac{\mathrm{d}^{1/2}}{\mathrm{d}(-t)^{1/2}}$ is the anti-causal time half-derivative of the input traces $U(\xi,t) = U_0(\xi)F(t-\tau_\mathrm{R}(\xi))$ recorded in the geophone (receiver) $\mathbf{g}(\xi)$, corresponding to a punctual source $\mathbf{s}(\xi)$ (see [6]), $W_{DS}^{(2.5\mathrm{D})}$ the migration weigh function and τ_D the migration stacking curve. The asymptotical evaluation (1), through the method of stationary phase (see [1]) along with the medium special characteristics allows expressing the weight function for vertically inhomogeneous medium through the formula (see [5]),

$$W_{DS}^{(2.5\mathrm{D})}(\xi,\mathbf{m}) = c(\mathbf{m})L_{\mathbf{s}}^{(2\mathrm{D})}L_{\mathbf{g}}^{(2\mathrm{D})}\sqrt{(\sigma_{\mathbf{s}}+\sigma_{\mathbf{g}})}\left[\frac{\cos\alpha_{\mathbf{m}}^{\mathbf{s}}}{\sigma_{\mathbf{s}}}+\frac{\cos\alpha_{\mathbf{m}}^{\mathbf{g}}}{\sigma_{\mathbf{g}}}\right], \tag{2}$$

where $c(\mathbf{m})$ is the speed in a known in-depth point $\mathbf{m}$, σ_i, and $\alpha_{\mathbf{m}}^i$, and $L_i^{(2\mathrm{D})}$, $(i = \mathbf{s},\mathbf{g})$ the ray parameter, the angle between the ray and the depth in $\mathbf{m}$ and the 2D geometric spreading, respectively, (see [5]).

The migration stacking-curve for this case is established through the expression

$$\tau_D(\xi,\mathbf{m}) = \frac{1}{c_0}\left(\int_0^z \frac{n^2(z')\,\mathrm{d}z'}{\sqrt{n^2(z')-\sin^2\alpha_0^{\mathbf{s}}}} + \int_0^z \frac{n^2(z')\,\mathrm{d}z'}{\sqrt{n^2(z'-\sin^2\alpha_0^{\mathbf{g}})}}\right). \tag{3}$$

3. Vertically inhomogeneous medium

The analytical expressions for the horizontal distance between sources-and-receivers, for the ray parameter and transit time are, respectively (see [5]):

$$x - x_i = \sin\alpha_0^i \int_0^z \frac{\mathrm{d}z'}{\sqrt{n^2(z')-\sin^2\alpha_0^i}}, \quad (i = \mathbf{s},\mathbf{g}), \tag{4}$$

$$\sigma_i = c_0 \int_0^z \frac{\mathrm{d}z'}{\sqrt{n^2(z')-\sin^2\alpha_0^i}}, \quad (i = \mathbf{s},\mathbf{g}), \tag{5}$$

$$\tau_i = \frac{1}{c_0}\int_0^z \frac{n^2(z')\,\mathrm{d}z'}{\sqrt{n^2(z')-\sin^2\alpha_0^i}}, \quad (i = \mathbf{s},\mathbf{g}). \tag{6}$$

It can be observed that for any medium with this type of velocity, a combination of expressions (4) and (5), shall give us

$$\sigma_i = \frac{c_0(x-x_i)}{\sin\alpha_0^i} \quad (i = \mathbf{s},\mathbf{g}). \tag{7}$$

On the other hand, the 2-D geometric spreading factor is given by

$$L_i^{(2\mathrm{D})} = \left[\frac{\cos\alpha_0^i\cos\alpha_{\mathbf{m}}^i}{c_0}\int_0^z \frac{n^2(z')\,\mathrm{d}z'}{(n^2(z')-\sin^2\alpha_0^i)^{3/2}}\right]^{1/2}. \tag{8}$$

3.1. Stacking curve

Finally, the expression which specifies the transit time may be written as (see [5]),

$$\tau_D = \frac{1}{c_0}\left[\int_0^z \frac{n^2(z')\,\mathrm{d}z'}{(n^2(z') - \sin^2\alpha_0^{\mathbf{s}})^{1/2}} + \int_0^z \frac{n^2(z')\,\mathrm{d}z'}{(n^2(z') - \sin^2\alpha_0^{\mathbf{g}})^{1/2}}\right], \tag{9}$$

where $\alpha_0^{\mathbf{s}}(\alpha_0^{\mathbf{g}})$ is the angle between the z-axis and the ray connecting the source $\mathbf{s}$ (receiver $\mathbf{g}$) to the point $\mathbf{m}$ over the reflector, i.e., with the vertical component z.

3.2. Weight function

The migration weight function for this medium type may be written as (see [5])

$$W_{DS}^{(2.5\mathrm{D})}(\xi,\mathbf{m}) = c(\mathbf{m})L_{\mathbf{s}}^{(2\mathrm{D})}L_{\mathbf{g}}^{(2\mathrm{D})}\sqrt{(\sigma_{\mathbf{s}}+\sigma_{\mathbf{g}})}\left[\frac{\cos\alpha_{\mathbf{m}}^{\mathbf{s}}}{\sigma_{\mathbf{s}}} + \frac{\cos\alpha_{\mathbf{m}}^{\mathbf{g}}}{\sigma_{\mathbf{g}}}\right], \tag{10}$$

where $c(\mathbf{m})$ represents the velocity at the depth point $\mathbf{m}$, and the quantities σ_i, $\cos\alpha_{\mathbf{m}}^i$, $L_i^{(2\mathrm{D})}$ $(i = \mathbf{s},\mathbf{g})$ are described by the expressions (5), (7) and (8), respectively.

4. Analytical cases

In this section we intend to establish analytical expressions for stacking curves weight functions for three specific cases where velocity depends on depth (constant gradient slowness, constant gradient of quadratic velocity and constant gradient of cubic velocity, see illustration in Fig. 1). These expressions allow the migration integral to be analytically solved. Fig. 2–7 illustrate the stacking curves and weight functions for these models.

5. Constant gradient slowness

Substituting the velocity model

$$\frac{1}{c(z)} = \frac{1}{c_0} + gz, \tag{11}$$

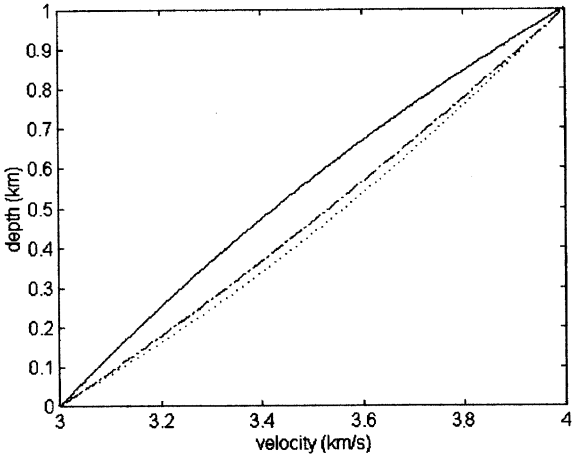

Fig. 1. Examples of three distributions: constant gradient of cubic velocity (dotted line), constant gradient slowness (continuous line) and constant gradient of quadratic velocity (dotted-dashed line).

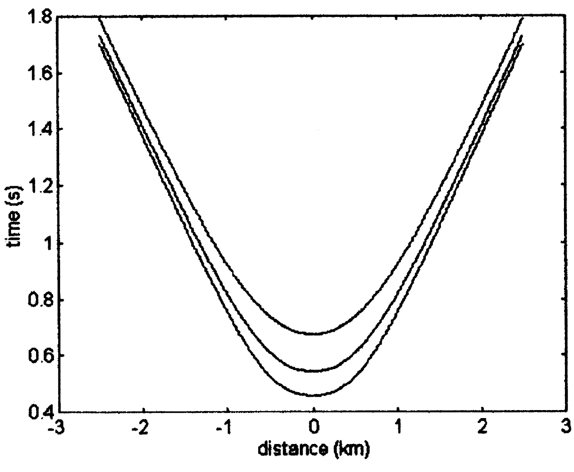

Fig. 2. Migration stacking curve with slowness and constant gradient.

in (5), (6) and (8), we have for $i = \mathbf{s}, \mathbf{g}$, respectively

$$x - x_i = \frac{\sin \alpha_0^i \sqrt{\ln^2[1 + gzc_0] + g^2c^2(x - x_i)^2}}{gc_0}, \tag{12}$$

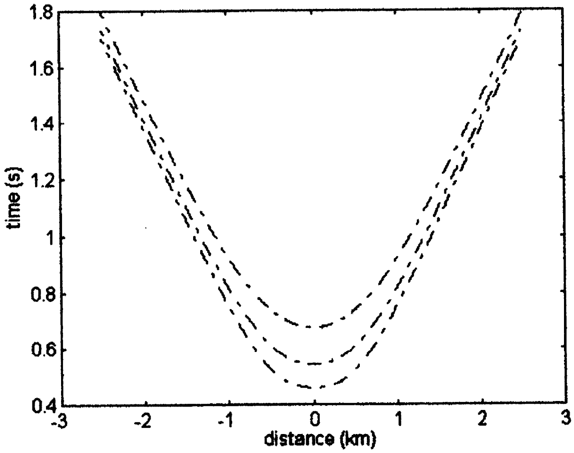

Fig. 3. Migration stacking curve for a constant gradient of quadratic velocity.

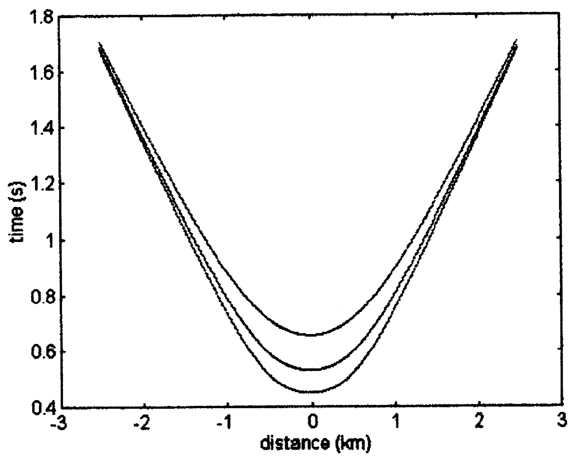

Fig. 4. Migration stacking curve for a constant gradient of cubic velocity.

$$\sigma_i = \frac{\sqrt{\ln^2[1 + gzc_0] + g^2c^2(x - x_i)^2}}{g}, \tag{13}$$

and

$$\overline{L}_i^{2D} = \frac{\sqrt[4]{\ln^2(1 + gzc_0) + g^2(x - x_i)^2(c^2 - c_0^2)}\sqrt{\ln(1 + gzc_0)}}{gc_0 \cos\alpha_{\mathbf{m}}^i \sqrt{\sigma_i}}. \tag{14}$$

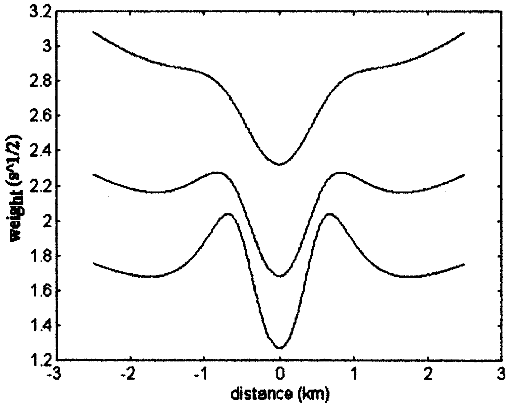

Fig. 5. Migration weight function for a constant gradient slowness.

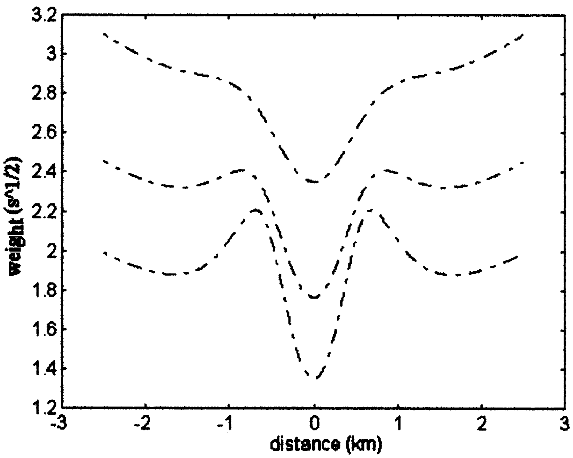

Fig. 6. Migration weight function for a constant gradient of quadratic velocity.

The expressions (12)–(14) are known as horizontal distances between sources-and-receivers, the parameter taken along the **sg** ray and the geometric spreading integrant term, respectively.

5.1. Stacking curve

The stacking curve is established by substituting the velocity distribution (11) into integral Eq. (9). After some algebraic manipulation, it can be written as

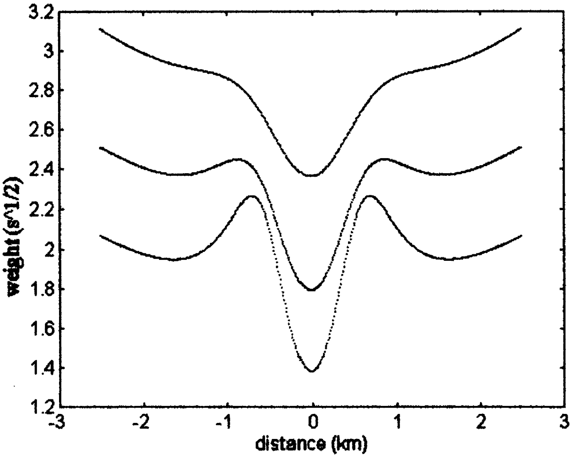

Fig. 7. Migration weight function for a constant gradient of cubic velocity.

$$\tau_D(\xi,\mathbf{m}) = \frac{(2z+gc_0z^2)}{2c_0\ln[1+gzc_0]}\left[\sqrt{\ln^2[1+gzc_0]+g^2c^2(x-x_\mathbf{s})^2} + \sqrt{\ln^2[1+gzc_0]+g^2c^2(x-x_\mathbf{g})^2}\right]. \tag{15}$$

When writing (15) using expressions (12) and (13), we have

$$\tau_D(\xi,\mathbf{m}) = \frac{gz(c+c_0)(\sigma_\mathbf{s}+\sigma_\mathbf{g})}{2cc_0\ln[1+gzc_0]}. \tag{16}$$

The function (16) is the migration stacking curve with velocity distribution given by (11), as observed in Fig. 2 for same depth points.

5.2. Weight function

The migration weight function is established by substituting expressions (13) and (14) into Eq. (10). The result is given by

$$W_{DS}(\xi,x,z) = \frac{cP_\mathbf{s}P_\mathbf{g}}{gc_0^2}\left(\frac{\sigma_\mathbf{s}}{\sigma_\mathbf{g}}+\frac{\sigma_\mathbf{g}}{\sigma_\mathbf{s}}\right)\sqrt{\frac{1}{\sigma_\mathbf{s}}+\frac{1}{\sigma_\mathbf{g}}}, \tag{17}$$

where

$$P_i = \sqrt[4]{[\ln^2(1+gzc_0)]+g^2(x-x_i)^2(c^2-c_0^2)} \quad (i=\mathbf{s},\mathbf{g}).$$

The expression (17) is the migration weight function with velocity distribution (11). This weight function is depicted in Fig. 5.

6. Constant gradient of quadratic velocity

Substituting the velocity distribution

$$c^2(z) = c_0^2 + gz, \tag{18}$$

into (5), (6) and (8), we produce,

$$x - x_i = \frac{2\sin\alpha_0^i[c^3 - c_0^3]}{3gc_0\cos\alpha_{\mathbf{m}}^i}, \quad \sigma_i = \frac{\sqrt{4[c^3 - c_0^3]^2 + 9g^2c^2(x - x_i)^2}}{3g}, \tag{19}$$

and

$$\overline{L}_i^{2\mathrm{D}} = \frac{\sigma_i\sqrt{3g\cos\alpha_0^i}}{c_0\sqrt{2}\sqrt{c^3 - c_0^3}}, \tag{20}$$

where

$$\cos\alpha_{\mathbf{m}}^i = \frac{2[c^3 - c_0^3]}{\sqrt{4[c^3 - c_0^3] + 9g^2c^2(x - x_i)^2}}, \tag{21}$$

and

$$\sin^2\alpha_0^i = \frac{9g^2c_0^2(x - x_i)^2}{[4c^3 - c_0^3]^2 + 9g^2c^2(x - x_i)^2} \quad \text{for}(i = \mathbf{s}, \mathbf{g}). \tag{22}$$

The expressions (19), (20) are known as horizontal distances between sources-and-receivers, the parameter taken along the **sg** ray and the geometric spreading integrant term, respectively.

6.1. Stacking curve

The stacking curve is established substituting the velocity model (18) into integral (9). The result may be written as

$$\tau_D(\xi, \mathbf{m}) = \frac{3[\sigma_{\mathbf{s}} + \sigma_{\mathbf{g}}]}{(c^2 + cc_0 + c_0^2)}. \tag{23}$$

The function (23) is the migration stacking curve with the velocity model (18). This migration stacking curve is depicted in Fig. 3.

6.2. Weight function

The migration weight function is established by substituting expressions (19) and (20) into Eq. (10). The result is given by

$$W_{DS}(\xi,x,z) = \frac{cQ_{\mathbf{s}}Q_{\mathbf{g}}}{3gc_0^2}\left(\frac{\sigma_{\mathbf{s}}}{\sigma_{\mathbf{g}}}+\frac{\sigma_{\mathbf{g}}}{\sigma_{\mathbf{s}}}\right)\sqrt{\frac{1}{\sigma_{\mathbf{s}}}+\frac{1}{\sigma_{\mathbf{g}}}}, \tag{24}$$

where

$$Q_i = \sqrt[4]{[4(c^3-c_0^3)^2 + 9g^2(x-x_i)^2(c^2-c_0^2)]} \quad i = (\mathbf{s},\mathbf{g}).$$

The expression (24) is the weight function for the velocity distribution (18). This function is depicted in Fig. 6 for the same depth points.

7. Constant gradient of cubic velocity

We now substitute the velocity model

$$c^3(z) = c_0^3 + gz \tag{25}$$

into (5), (6) and (8), which gives us

$$x - x_i = \frac{3\sin\alpha_0^i[c^4-c_0^4]}{4gc_0\cos\alpha_{\mathbf{m}}^i}, \tag{26}$$

where

$$\cos\alpha_{\mathbf{m}}^i = \frac{3[c^4-c_0^4]}{\sqrt{9[c^4-c_0^4]^2+16c^2g^2(x-x_i)^2}}, \quad \sin^2\alpha_0^i = \frac{c_0^2(x-x_i)^2}{\sigma_i^2}, \tag{27}$$

$$\sigma_i = \frac{\sqrt{9[c^4-c_0^4]^2+16g^2c^2(x-x_i)^2}}{4g}e, \quad \overline{L}_i^{2D} = \frac{\sqrt{3}\sqrt{\cos\alpha_0^i}\sqrt{(c^4-c_0^4)}}{2c_0\cos\alpha_{\mathbf{m}}^i\sqrt{g}} \tag{28}$$

with $i = \mathbf{s},\mathbf{g}$.

7.1. Stacking curve

The stacking curve is established substituting the velocity model (25) into integral (9). The result is given by

$$\tau_D(\xi,\mathbf{m}) = \frac{2[\sigma_{\mathbf{s}}+\sigma_{\mathbf{g}}]}{[c^2+c_0^2]}. \tag{29}$$

The function (29) is the migration stacking curve for the velocity model (25). The migration stacking curve is depicted in Fig. 4.

7.2. Weight function

The migration weight function is established substituting expressions (28) into Eq. (10). The result is

$$W_{DS}(\xi,x,z)=\frac{cT_{\mathbf{s}}T_{\mathbf{g}}}{4gc_0^2}\left(\frac{\sigma_{\mathbf{s}}}{\sigma_{\mathbf{g}}}+\frac{\sigma_{\mathbf{g}}}{\sigma_{\mathbf{s}}}\right)\sqrt{\frac{1}{\sigma_{\mathbf{s}}}+\frac{1}{\sigma_{\mathbf{g}}}}, \tag{30}$$

where

$$T_i=\sqrt[4]{[9(c^4-c_0^4)^2+16g^2(x-x_i)^2(c^2-c_0^2)]}\quad (i=\mathbf{s},\mathbf{g}).$$

The expression (30) is the migration weight function for the velocity distribution (25). Fig. 7 illustrates some depth points.

7.3. Comparison

Aiming to compare the obtained results, the stacking curves and weight functions for the three velocity cases, which were object of analytical study in the previous sections, are illustrated in Figs. 8–11. The results are compared taking into account depth point at $z = 1$ km and velocity of 4 km/s for all cases. It may be observed that, despite the relative difference in the analytical expressions of the stacking curves, all results have nearly the same appearance (hyperbolas).

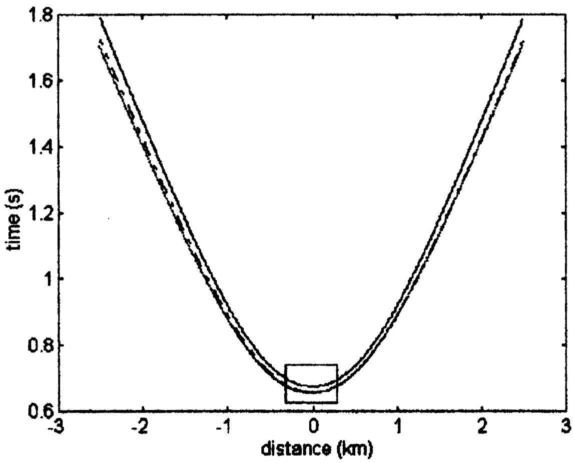

Fig. 8. Comparison of the migration stacking curves for depth point $z = 1$ km for a constant gradient slowness (continuous line), for a constant gradient of quadratic velocity (dotted-dashed line) and for a constant gradient of cubic velocity (dotted line).

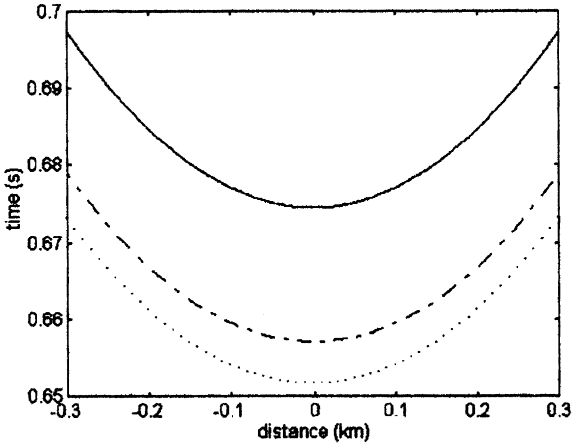

Fig. 9. Detailed view of box indicated in Fig. 8.

Figs. 2–4 contemplate the migration stacking curves for constant gradient slowness, constant gradient of quadratic velocity, and constant gradient of cubic velocity at depth points $z = 0.5$, 0.7 and 1 km.

Figs. 5–7 illustrate weight functions for the three cases of velocity with vertical dependence taking into account the same depth points previously used.

Fig. 8 shows that constant gradient of cubic velocity have shorter arrival times, followed by constant gradient of quadratic velocity it also shows that the constant gradient slowness presents longer times than the previous models.

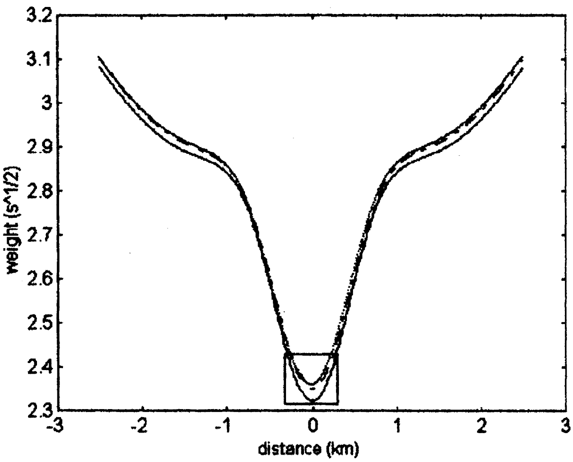

Fig. 10. Comparison of the migration weight functions for a depth point at $z = 1$ km for a constant gradient slowness (continuous line), for a constant gradient of quadratic velocity (dotted-dashed line) and for a constant gradient of cubic velocity (dotted line).

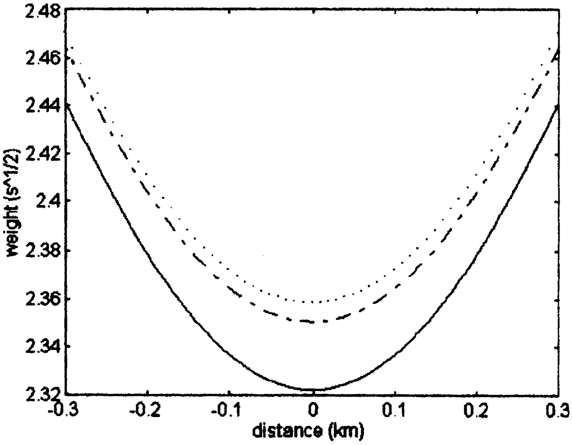

Fig. 11. Detailed view of box indicated in Fig 10.

Fig. 9 shows a detailed picture of this fact (the box indicated in Fig. 8 in a closer view). Fig. 10 illustrates the weight function for the three cases where velocity depends on depth point, with $z = 1$ km. Finally, Fig. 11 shows the zoomed box depicted in Fig. 10. It has been also observed that constant gradient slowness has lower weight, followed by constant gradient quadratic velocity. By its turn, constant gradient cubic velocity has, at the same point, a higher weight.

8. Final considerations

In this work we have presented a class of results corresponding to migration stacking curves and migration weight functions for some distribution of vertical velocities. The velocity distributions were chosen in such way that the slowness, quadratic and cubic velocities had constant vertical gradient. The choice of these velocity distributions is due to the fact that they allow the use of analytic formulas for the quantities involved in ray tracing [1]. So, they also allow analytic tracing of the imaging operation. For the numeric illustration, we have used very similar velocity models for the three velocity distribution, i.e., velocities coincident at $z = 0$ (velocity of 3 km/s) and at $z = 1$ km (velocity of 4 km/s). It is worth to observe that these models were chosen in such manner that these characteristics may come to be similar. Hence, we hope the numeric implementations for the analytic formulas pertaining to weight functions and stacking curves are also similar. This way, eventual errors in the development of formulas and its numeric implementation through the comparison of results for these three special cases may be promptly observed. Obviously, in other models, the numeric values of these analytical formulas may present significant differences, for example, if velocities were taken having the same vertical gradient at the beginning of the interval, but thy diverge more and more as depth increases. It is important to study these special cases, because they can be used to approximate more complex velocity distributions. Because the respective analytical formulas are known, faster calculation of the quantities corresponding to ray tracing theory approximations of this kind are often used in practice, mostly when the need of faster results at lower computational cost is a essential. In these circumstances, the use of these velocity distributions often provides very satisfactory results for the depth section being studied. It is worth to observe the analytical formulas obtained can also be used to minimize the computational cost for dealing with more complex situations, as they may be sufficient to represent any velocity distribution by a set of layers or cells for each one of these distributions. Thus, by adjusting the boundary conditions to allow the connection of these layers or cells, it is possible to create a ray tracing to determine the desired quantities, cell by cell, just by using known analytical formulas.

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