Attenuation of the intensity within a superdeformed band

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Abstract

The attenuation of the intra-band intensity of a superdeformed band which results from mixing with normally deformed configurations is calculated using reaction theory. It is found that the sharp increase of the attenuation is mostly due to the tunnelling through a spin dependent barrier and not to the chaotic nature of the normally deformed states.

It is now well established that the intensities of $E2$ gamma transitions within a superdeformed (SD) rotational band show cascades down to low angular momentum [1–7]. These cascades exhibit the distinct feature that the intensity remains constant until a certain spin is reached where-after the intensity drops to zero within a few transitions. The sharp drop in intensity is commonly referred to as the decay out of a superdeformed band and is believed to arise from mixing of the SD states with normally deformed (ND) states of identical spin.

The earliest theoretical work to implement such an interpretation [8–11] used a statistical model of the coupling between the SD and ND states. More recently, Refs. [12,13] used a framework originally developed for the study of compound nuclear reactions to derive formulae for the intensity in a more rigorous fashion (the expressions for the intensity in Refs. [8–11] are deduced from probability arguments). Ref. [13] concluded that Refs. [8–11] are valid in the non-overlapping resonance region. Refs. [8–11] further calculate the spin dependence of the relevant parameters (the electromagnetic widths of the SD and ND states, the level density of the ND states and spin dependence of the barrier separating the SD and ND wells) which Refs. [12,13] do not. Two features common to Refs. [8–13] are (i) the use of the Gaussian Orthogonal Ensemble (GOE) to simulate the ND states (ii) the use of the "golden rule" to extract a width for the the SD states due to mixing with the ND states.

Here, as in Refs. [12,13] we exploit the similarity between the decay out of superdeformed bands and compound nuclear reactions to write the intensity as the sum of average and fluctuation contributions. However we use an energy average in place of the ensemble average used in Refs. [12,13]. The energy average approach allows the inclusion of the following features which are more difficult to incorporate into an ensemble average.
(i) A hierarchy of complexity in the ND spectrum may be introduced.
(ii) A statistical model different from the GOE may be used to simulate the ND states, as was proposed in Refs. [14,15].
(iii) A width for the SD states due to mixing with the ND states arises naturally without appealing to the "golden rule" whose range of validity has been found to be restricted [16,17].

In Figure 1 we show a schematic plot of the energy of ND and SD bands as a function of spin. The observable in which attenuation is seen is the total intensity of two consecutive $E2$ photons in the cascade down the SD band. Let $|J\rangle$ denote an SD configuration with spin J. The relative intensity of the two step transition $|J+2\rangle \stackrel{\gamma_1}{\rightarrow} |J\rangle \stackrel{\gamma_2}{\rightarrow} |J-2\rangle$ (relative to the intensity of the same two step transition in the absence of mixing with other configurations) is given by

$$\begin{aligned} F_J &= \frac{1}{2\pi\Gamma^{\gamma}_{J+2}} \int_{-\infty}^{\infty} dE_{\gamma_1} \int_{-\infty}^{\infty} dE_{\gamma_2} |\langle J-2|T(E_{J-2}+E_{\gamma_2})|J+2\rangle|^2 \delta(E - E_{J-2} - E_{\gamma_2}) \\ &= \frac{1}{2\pi\Gamma^{\gamma}_{J+2}} \int_{-\infty}^{\infty} dE |\langle J-2|T(E)|J+2\rangle|^2, \end{aligned} \tag{1}$$

where $E \equiv E_{J+2} - E_{\gamma_1} = E_J$ takes account of the Hamiltonian of the electromagnetic field, E_J being the energy of $|J\rangle$ and E_{γ_1} and E_{γ_2} the energies of the two consecutive photons. The electromagnetic width of $|J+2\rangle$ is Γ^{γ}_{J+2} making $2\pi\Gamma^{\gamma}_{J+2}$ the intensity when there is no mixing with the ND states and thus no flux loss from the SD band. Note that in Eq. (1) we ignore the widths of the initial and final states for the purpose of calculating the relative intensity.

The transition amplitude is given by

$$\langle J-2|T(E)|J+2\rangle = \gamma_{J+2} \langle J|G(E)|J\rangle \gamma_J. \tag{2}$$

Here γ_{J+2} is the electromagnetic decay amplitude of $|J+2\rangle$ defined such that $\Gamma^{\gamma}_{J+2} = \gamma^2_{J+2}$; γ_J and $\Gamma^{\gamma}_J = \gamma^2_J$ are the corresponding decay amplitude and width of $|J\rangle$ whilst the Green's function is given by

$$G(E) = (E - H)^{-1}. \tag{3}$$

The total nuclear Hamiltonian, which takes the coupling to the electromagnetic field into account, is denoted by H.

The projected Green's function $\langle J|G(E)|J\rangle$ may be expressed in terms of its Lorentzian energy average $\langle J|G^{\text{av.}}(E)|J\rangle = \langle J|G(E + \frac{iI}{2})|J\rangle$ (energy averaging interval I) plus a fluctuation part [18,19]:

$$\langle J|G|J\rangle = \langle J|G^{\text{av.}}|J\rangle + \langle J|G^{\text{fluc.}}|J\rangle, \tag{4}$$

where by definition the energy average of $\langle J|G^{\text{fluc.}}|J\rangle$ is zero. Thus Eq. (1) for the relative intensity may be written

$$F_J = F^{\text{av.}}_J + F^{\text{fluc.}}_J, \tag{5}$$

where

$$F^{\text{av.}}_J = \frac{\Gamma^{\gamma}_J}{2\pi} \int_{-\infty}^{\infty} dE |\langle J|G^{\text{av.}}|J\rangle|^2 \tag{6}$$

and

$$F^{\text{fluc.}}_J = \frac{\Gamma^{\gamma}_J}{2\pi} \int_{-\infty}^{\infty} dE \left(|\langle J|G^{\text{fluc.}}|J\rangle|^2 \right)^{\text{av.}}. \tag{7}$$

In this paper we focus our discussion on $F_J^{\text{av.}}$. It can be shown that

$$\langle J|G^{\text{av.}}(E)|J\rangle = \frac{1}{E - E_J + i\Gamma_J^\gamma/2 - W_{JJ}(E)}. \tag{8}$$

The derivation of Eq. (8) for $\langle J|G^{\text{av.}}|J\rangle$ and an expression for $\langle J|G^{\text{fluc.}}|J\rangle$ using projection operator techniques will be reported in a subsequent paper.

The form of $W_{JJ}(E)$ depends on the specific model for the Hamiltonian which is employed. It is our aim to study whether or not the chaotic nature (as classified by random matrix theory [RMT]) of the ND states is decisive in explaining the observed attenuation. In order to isolate the statistical aspects of the calculation we use two different models distinguished by whether the tunnelling interaction mixes $|J\rangle$ randomly with the ND states (*model A*) or whether it couples more strongly to certain ND states than others (*model B*). In the latter we shall make the most extreme assumption that $|J\rangle$ couples to only one ND state.

Model A is represented by the matrix

$$H \to \begin{pmatrix} E_J & V_{Jn} \\ V_{Jn} & E_n\delta_{n'n} \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_J^\gamma & 0 \\ 0 & \Gamma_N^\gamma\delta_{n'n} \end{pmatrix}, \quad n = 1, ..., N, \tag{9}$$

where E_n denotes the energies of the N ND states with which $|J\rangle$ mixes due to the real tunnelling interaction V_{Jn}. Here Γ_N^γ is an electromagnetic width which we assume to be common to the ND states. With these definitions $W_{JJ}(E)$ becomes

$$W_{JJ}(E) = \sum_{n=1}^{N} \frac{[V_{Jn}]^2}{E - E_n + i\left(\Gamma_N^\gamma + I\right)/2}. \tag{10}$$

The energies E_n are constructed using the deformed Gaussian orthogonal ensemble (DGOE) [20]. The DGOE allows a smooth interpolation from Poisson to GOE statistics by varying a mixing parameter λ from 0 to 1. Thus the E_n are the eigenvalues of a random Hamiltonian h which is real symmetric and whose matrix elements are taken to be Gaussian distributed random numbers with zero mean and variances

$$\langle h_{nn}^2\rangle = \frac{2}{N}, \quad \langle h_{n'n}^2\rangle = \frac{\lambda^2}{N}, n' \neq n. \tag{11}$$

The random tunnelling interaction is taken to have zero mean and variance

$$\langle V_{Jn}^2\rangle = v_J^2. \tag{12}$$

We assume that E_J lies in the middle of the N ND states, that is, $E_J = 0$. Following Refs. [8,9,12,13] we introduce a spreading width $\Gamma_J^\downarrow$ through the golden rule

$$\Gamma_J^\downarrow = 2\pi v_J^2. \tag{13}$$

We maintain doubts about the meaningfulness of Eq. (13) regarding its interpretation as a width [17]. For practical purposes, however, it is a change of variable from v_J to $\Gamma_J^\downarrow$. All quantities of dimensions energy are to be understood to have units of D_J, where D_J denotes the mean spacing of the ND states around E_J. Thus the E_n (and the E_q and $\mathcal{V}_{dq}$ in

Eq. (15)) which are generated from the DGOE are to be understood to have been unfolded such that the E_n and E_q have mean spacing equal to unity. Thus we may write $W_{JJ}(E)$ as

$$W_{JJ}(E) = \frac{\Gamma_J^{\downarrow}}{2\pi} \sum_{n=1}^{N} \frac{g_n^2}{E - E_n + i\left(\Gamma_N^{\gamma} + I\right)/2}. \tag{14}$$

where the g_n, $n = 1, .., N$, are Gaussian distributed random numbers with zero mean and unit variance.

Model B is represented by the matrix

$$H \rightarrow \begin{pmatrix} E_J & V_{Jd} & 0 \\ V_{Jd} & E_d & \mathcal{V}_{dq} \\ 0 & \mathcal{V}_{dq} & E_q \delta_{q'q} \end{pmatrix} - \frac{i}{2} \begin{pmatrix} \Gamma_J^{\gamma} & 0 & 0 \\ 0 & \Gamma_N^{\gamma} & 0 \\ 0 & 0 & \Gamma_N^{\gamma} \delta_{q'q} \end{pmatrix}, \quad q = 1, ..., N-1. \tag{15}$$

We assume here that $|J\rangle$ couples to only one state of normal deformation; $|d\rangle$, which has energy E_d; with strength V_{Jd}. This special state is subsequently mixed with other ND configurations with energies E_q by a residual interaction $\mathcal{V}_{dq}$. Now $W_{JJ}(E)$ becomes

$$W_{JJ}(E) = \left[V_{Jd}\right]^2 \sum_{n=1}^{N} \frac{\left[c_d(n)\right]^2}{E - E_n + i\left(\Gamma_N^{\gamma} + I\right)/2} \tag{16}$$

where $c_d(n)$ denotes component d of the nth eigenvector of the sub-matrix of the first term of Eq. (15) obtained by excluding the first row and the first column. Now the E_q are eigenvalues of a random Hamiltonian h which is real symmetric and whose matrix elements are taken to be Gaussian distributed random numbers with zero mean and variances

$$\langle h_{qq}^2 \rangle = \frac{2}{N}, \quad \langle h_{q'q}^2 \rangle = \frac{\lambda^2}{N}, q' \neq q. \tag{17}$$

The residual interaction is also taken to have zero mean and variance

$$\langle \mathcal{V}_{dq}^2 \rangle = \frac{\lambda^2}{N}. \tag{18}$$

We put $E_d = E_J = 0$. Introducing

$$\Gamma_J^{\downarrow} = 2\pi \frac{\left[V_{Jd}\right]^2}{N}, \tag{19}$$

we can write Eq. (16) as

$$W_{JJ}(E) = \frac{\Gamma_J^{\downarrow}}{2\pi} \sum_{n=1}^{N} \frac{N\left[c_d(n)\right]^2}{E - E_n + i\left(\Gamma_N^{\gamma} + I\right)/2}. \tag{20}$$

Thus comparing Eq. (14) with Eq. (20) we see that, although the meaning of $\Gamma_J^{\downarrow}$ is different for the two models, the difference between *model A* and *model B* boils down how much the distribution $\left[c_d(n)\right]^2$ differs from that of g_n^2. This difference is not trivial as $\left[c_d(n)\right]^2$ has a dramatic λ dependence (see Fig. (1) in Ref. [15]). The inclusion of the factor $\frac{1}{N}$ in Eq. (19) makes clear that *model A* and *model B* are only comparable when V_{Jd}^2 is of the order Nv_J^2.

Model A is precisely equivalent to that of Refs. [12,13] when $\lambda = 1$. The real part of Eq. (15) used in *model B* is equivalent to what is used in Refs. [14,15], however, we calculate the average intensity integrated over the energy: $F_J^{\text{av.}}$, whereas Refs. [14,15] calculate a tunnelling probability which is more closely related to $W_{JJ}(E)$.

Note that $F_J^{\text{av.}}$ can be written as

$$F_J^{\text{av.}} = \frac{\Gamma_J^\gamma}{2\pi}\int_{-\infty}^{\infty} dE \frac{1}{(E - E_J - \Delta_J^\downarrow(E))^2 + \frac{\Gamma_J^{\gamma 2}}{4}(1 + \frac{\Gamma_J^\downarrow(E)}{\Gamma_J^\gamma})^2}, \tag{21}$$

where

$$\Delta_J^\downarrow(E) = \text{Re}W_{JJ}(E), \tag{22}$$

and

$$\Gamma_J^\downarrow(E) = -2\text{Im}W_{JJ}(E). \tag{23}$$

Ignoring the shift $\Delta_J^\downarrow(E)$ altogether and assuming that the width $\Gamma_J^\downarrow(E)$ has the energy independent value $\Gamma_J^\downarrow$ one obtains the principal result of Ref. [12] that

$$F_J^{\text{av.}} \approx \frac{\Gamma_J^\gamma}{\Gamma_J^\gamma + \Gamma_J^\downarrow}. \tag{24}$$

We now present numerical calculations of $F_J^{\text{av.}}$ with $N = 50$, $\Gamma_J^\gamma = 0.01 D_J$. An ensemble average was performed over 100 realisations in Fig. (2) and over 1000 realisations in Fig (3). The effect of increasing Γ_N^γ, identical to that obtained by increasing the energy averaging interval I, is to broaden $\Gamma_J^\downarrow(E)$ (Eq. (23)) and thus push $F_J^{\text{av.}}$ closer to the approximation given by Eq. (24). This is in line with what it is reported in [21] who obtain Eq. (24) in the limit $\frac{\Gamma_N^\gamma}{\Gamma_S^\gamma} \to \infty$ for their two level model.

In our calculations we put $\Gamma_N^\gamma + I = 3D_J$. With this choice one may describe what Ref. [13] calls the overlapping resonance region. Ref. [13] gives the impression that the relative intensity is independent of Γ_J^γ. Whilst we agree with [13] that the ratios $b_J = \frac{\Gamma_J^\downarrow}{\Gamma_J^\gamma}$ and $\frac{\Gamma_N^\gamma}{D}$ are of principal importance in understanding the decay out, it can be seen from Eq. (21) that $F_J^{\text{av.}}$ is only independent of Γ_J^γ if $\Delta_J^\downarrow(E)$ and $\Gamma_J^\downarrow(E)$ are constant.

Fig. 2 shows the dependence of $F_J^{\text{av.}}$ on λ, the strength of the mixing amongst the ND states, for several values of b_J, for both *model A* and *model B*. For *model A* the variation of $F_J^{\text{av.}}$ with λ is rather slight compared to *model B*. This is because the λ dependence of *model A* is contained in the eigenvalues E_n which are unfolded to have unit mean spacing. *Model B* has a further and more significant λ dependence contained in the eigenvectors $c_d(n)$. For *model B*, $F_J^{\text{av.}}$ decreases with decreasing λ to a value which is limited by the value of $\Gamma_N^\gamma + I$. Note that $F_J^{\text{av.}}$ can change at most by a factor of about 5 by varying λ.

Fig. 3 shows $F_J^{\text{av.}}$ as function of b_J for some values of λ, calculated using *model B*. The calculations for *model A* are not shown as they can barely be distinguished from the calculation for $\lambda = 1$ using *model B*. The effect of changing λ is to move the value of b_J (and hence J) at which the decay out occurs. Thus from Fig. 3 we conclude that the decay out is slightly hindered by increasing λ.

Regarding Refs. [14,15] which report an increase in the tunnelling probability of several orders of magnitude with increasing λ we do not consider ourselves at odds with this work since, as already mentioned above, we do not calculate the same quantity. A further difference between *model B* of this paper and Refs. [14,15] is that their author places $|d\rangle$ at the position $\frac{N}{4}$ thus making the distribution $c_d(n)$ asymmetric. This would correspond in our calculation to making the difference $E_J - E_d$ non-zero (we see no reason not to set $E_J = E_d$).

An investigation of the roles of Γ_N^γ and I we postpone to a subsequent paper. The results of Ref [13] indicate that $F_J^{\text{fluc.}}$, Eq. (7), is important when Γ_N^γ is a small fraction of D_J (non-overlapping resonance region).

It was already found from phenomenological analysis some years ago [8,9,3] that $F_J^{\text{av.}}$ falls exponentially with decreasing spin. We conclude here that the chaotic nature of the ND states, as classified by λ, cannot account for such behaviour. The exponential drop in the intra-band intensity must be due to the spin dependence of the tunnelling matrix element contained in b_J. The calculation of b_J is not trivial and we refer the reader to [22,23], which continue the work of Refs. [8–11], for some recent calculations.

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REFERENCES

[1] P.J. Twin, B.M. Nyako, A.H. Nelson *et al.*, Phys. Rev. Lett. **57** (1986) 811.
[2] A. Atac *et al.*, Nucl. Phys. **A557** (1993) 109c.
[3] T. L. Khoo *et al.*, Nucl. Phys. **A557** (1993) 83c.
[4] R. Krücken *et al.* Phys. Rev. Lett. **73** (1994) 3359.
[5] R. Krücken *et al.* Phys. Rev. C **54** (1996) 1182.
[6] R. Kühn *et al.*, Phys. Rev. C **55** (1997) R1002.
[7] R. Krücken *et al.* Phys. Rev. C**55** (1997) R1625.
[8] E. Vigezzi, R. A. Broglia and T. Døssing, Phys. Lett. B **249** (1990) 163.
[9] E. Vigezzi, R. A. Broglia and T. Døssing, Nucl. Phys. **A520** (1990) 179c.
[10] Y. R. Shimizu, F. Barranco, R. A. Broglia, T. Dϕssing and E. Vigezzi, Phys. Lett. B **274** (1992) 253.
[11] Y.R. Shimuzu, E. Vigezzi, T. Dossing and R. Broglia, Nucl. Phys. **A557** (1993) 99c.
[12] H. A. Weidenmüller, P. von Brentano and B.R. Barrett, Phys. Rev. Lett. **81** (1998) 3603.
[13] Jian-zhong Gu and H. A. Weidenmüller Nucl. Phys. **A660** (1999) 197.
[14] S. Åberg, Phys.Rev. Lett. **82** (1999) 299.
[15] S Åberg, Nucl. Phys. **A649** (1999) 392c.
[16] N. Frazier, B.A. Brown and V. Zelevinsky, Phys. Rev. C **54** (1996) 1665.
[17] A. J. Sargeant, M. S. Hussein, M. P. Pato, M. Ueda, Phys. Rev. C **61** (2000) 011302R.
[18] M. Kawai, A.K. Kerman and K.W. McVoy, Ann. Phys. (N.Y.) **75** (1973) 156.
[19] H. Feshbach, *Theoretical Nuclear Physics: Nuclear Reactions* (Wiley, NY, 1992) Ch. IV, Sect. 8.
[20] M. S. Hussein and M.P. Pato, Phys. Rev. Lett. **70** (1993) 1089.
[21] C.A. Stafford and B.R. Barrett Phys. Rev. C **60** (1999) R051305.
[22] Y.R. Shimizu, M. Matsuo and K. Yoshida, Nucl. Phys. **A682** 464 (2001).
[23] K. Yoshida, M. Matsuo and Y.R. Shimizu, nucl-th/0010074, submitted to Nucl. Phys. A.

FIGURES

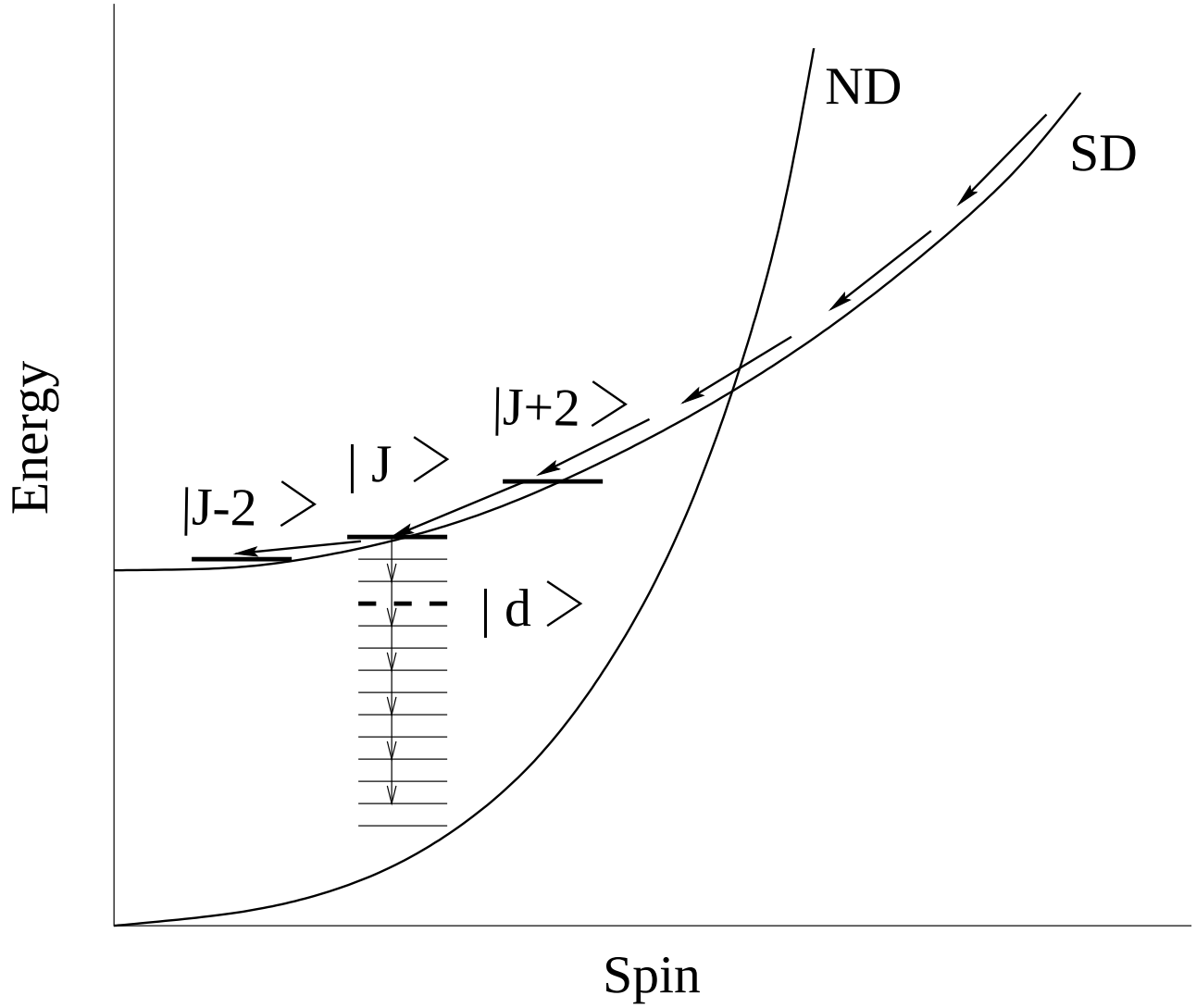

FIG. 1. Schematic diagram illustrating the decay out of a superdeformed band.

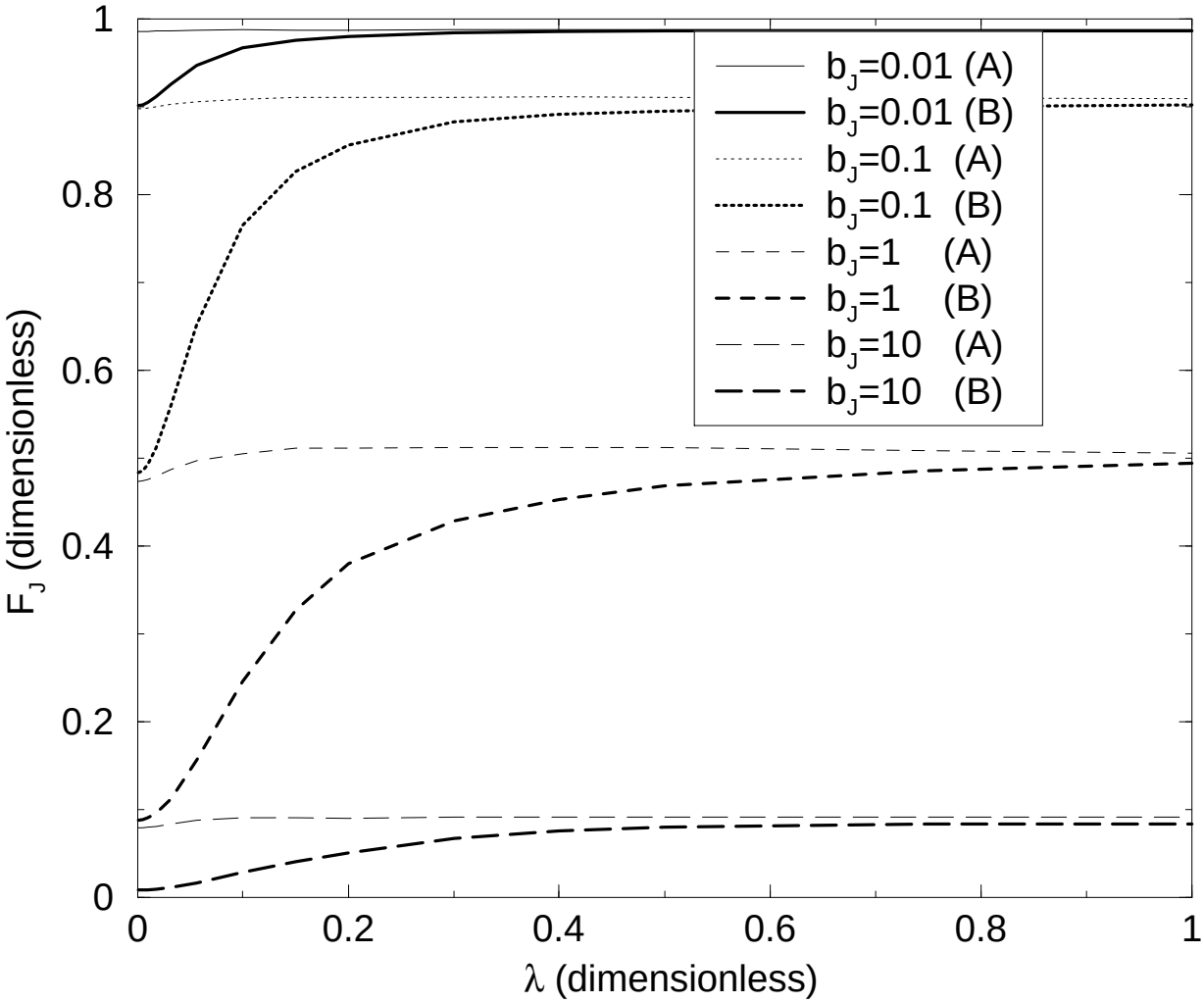

FIG. 2. Calculated attenuation factor $F_J^{\mathrm{av.}}$ as a function of λ for some values of $b_J = \frac{\Gamma_J^{\downarrow}}{\Gamma_J^{\gamma}}$. The thin lines where calculated using *model A* and the thick lines using *model B*.

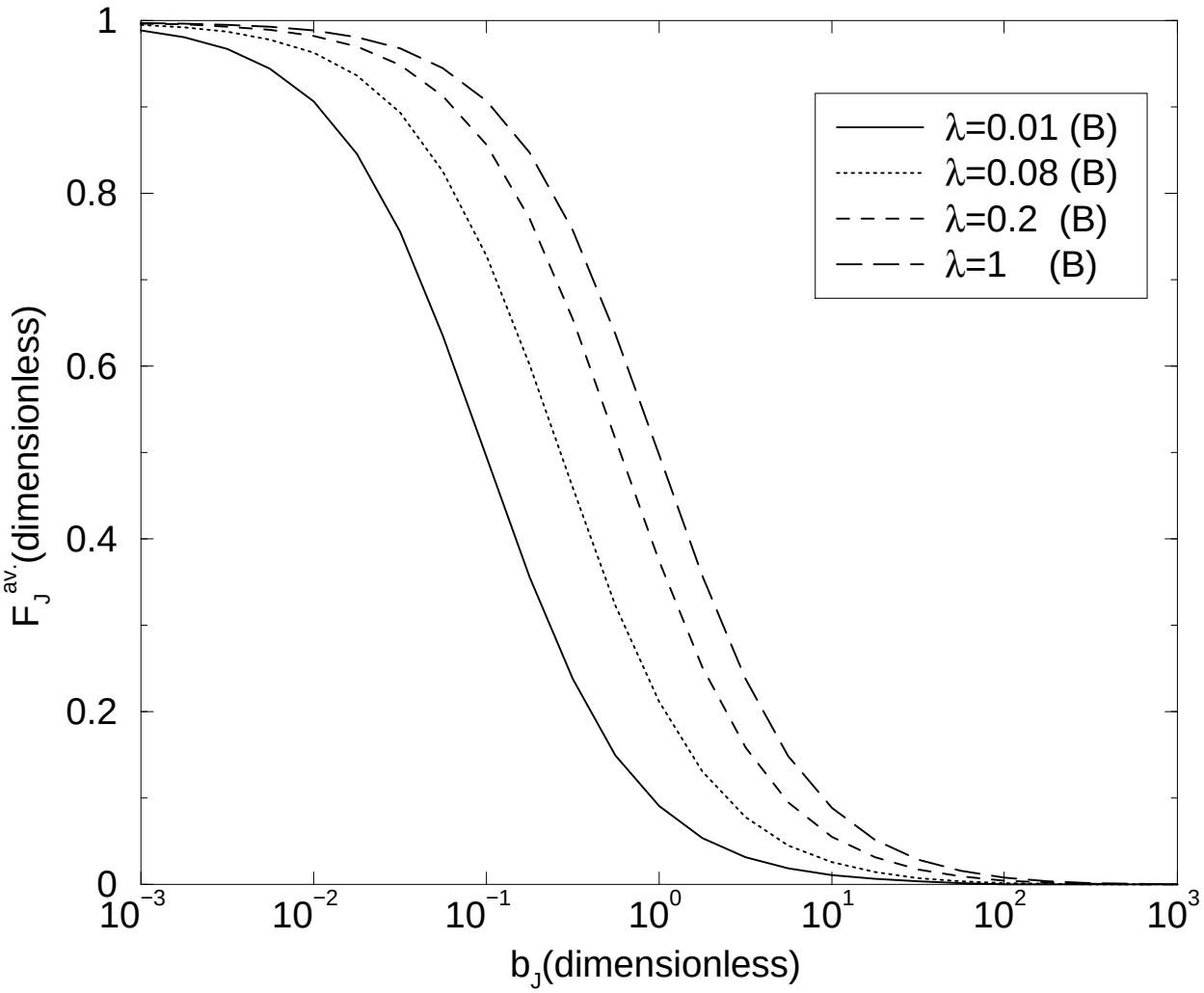

FIG. 3. Attenuation factor $F_J^{\mathrm{av.}}$ as a function of $b_J = \frac{\Gamma_J^{\downarrow}}{\Gamma_J^{\gamma}}$ for some values of λ, calculated using *model B*.