

## Addendum: Attenuation of the intensity within a superdeformed band

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We investigate a random matrix model [Phys. Rev. C **65**, 024302 (2002)] for the decay-out of a superdeformed band as a function of the parameters:  $\Gamma^\perp/\Gamma_S$ ,  $\Gamma_N/D$ ,  $\Gamma_S/D$ , and  $\Delta/D$ . Here  $\Gamma^\perp$  is the spreading width for the mixing of a superdeformed (SD) state  $|0\rangle$  with a normally deformed (ND) doorway state  $|d\rangle$ ,  $\Gamma_S$  and  $\Gamma_N$  are the electromagnetic widths of the SD and ND states, respectively,  $D$  is the mean level spacing of the compound ND states and  $\Delta$  is the energy difference between  $|0\rangle$  and  $|d\rangle$ . The maximum possible effect of a transition from ordered to chaotic states is inferred from analytical and numerical calculations of the decay intensity in the limiting cases for which the ND states obey Poisson and GOE statistics. Our results show that the sharp attenuation of the decay intensity cannot be explained solely by a transition from ordered to chaotic states.

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In superdeformed bands the total intraband decay intensity of the supercollective  $E2$   $\gamma$  transitions disappears suddenly due to tunneling through the barrier separating the superdeformed (SD) and normally deformed (ND) minima [1–4]. The theoretical calculation of the spin at which the decay-out occurs for different mass regions and the steepness of the attenuation of the decay intensity are subject to uncertainties concerning the density of ND states and the parameters describing the deformation barrier and collective motion [5]. In Ref. [6] Åberg suggested an alternative explanation of the sharp decay-out: a transition from ordered to chaotic states in the ND states enhances the tunneling probability and consequently the decay-out is a manifestation of “chaos assisted tunneling.”

In Ref. [7] the authors investigated Åberg’s suggestion by calculating the decay intensity as a function of the chaoticity parameter which produces a transition from order to chaos. We found that increasing the chaoticity did not enhance the decay-out and concluded on this basis that the decay-out must be due to the spin dependence of the barrier. Subsequently, Åberg [8] criticized our assumption of an energy difference of zero between the decaying SD state and the ND doorway state to which it is assumed to decay. In the following we study the decay intensity as a function of the energy difference and as a function of the other parameters relevant to the decay-out, calculating the decay intensity in the limits that the ND states obey Poisson and GOE statistics. This permits us to infer the maximum possible effect that an order-chaos transition in the ND states can have on the decay intensity. Our results reinforce our belief that the decay-out is mostly due to the spin dependence of the barrier.

The total average intraband decay intensity of an SD band is given by [7,12–15]

$$I_{\text{av}} = \frac{\Gamma_S}{2\pi} \int_{-\infty}^{\infty} dE \times \frac{1}{[E - E_0 - 2\pi|V_{0d}|^2 R_d]^2 + [\Gamma_S + 2\pi|V_{0d}|^2 S_d]^2/4}. \quad (1)$$

The intermediate SD state in the two-step decay which Eq. (1) describes is denoted  $|0\rangle$  and has energy  $E_0$ . The electromagnetic width for the intraband decay is  $\Gamma_S$ . In what follows we assume that  $|0\rangle$  only mixes (by tunneling through the barrier in deformation space separating the SD and ND wells) with one special ND doorway state  $|d\rangle$  whose energy is  $E_d$ . The interaction energy of  $|0\rangle$  and  $|d\rangle$  is  $V_{0d}$ . The state  $|d\rangle$  is subsequently mixed by the residual interaction with the remaining ND states,  $|Q\rangle$ ,  $Q=1, \dots, N$ , having the same spin as  $|0\rangle$  and  $|d\rangle$ . This strong-doorway assumption was called *model B* in Ref. [7]. The  $|Q\rangle$  lie in the interval  $L=ND$  where  $D$  denotes the mean spacing in energy of the  $|Q\rangle$ . The functions  $R_d(E)$  and  $S_d(E)$  describe the manner in which  $|d\rangle$  is distributed in energy over the remaining ND states and are given by

$$R_d(E) = \frac{1}{2\pi} \sum_{Q=0}^N |c_d(Q)|^2 \frac{E - E_Q}{(E - E_Q)^2 + \Gamma_N^2/4} \quad (2)$$

and

$$S_d(E) = \frac{1}{2\pi} \sum_{Q=0}^N |c_d(Q)|^2 \frac{\Gamma_N}{(E - E_Q)^2 + \Gamma_N^2/4}, \quad (3)$$

respectively, where  $\Gamma_N$  is the electromagnetic width of the ND states.

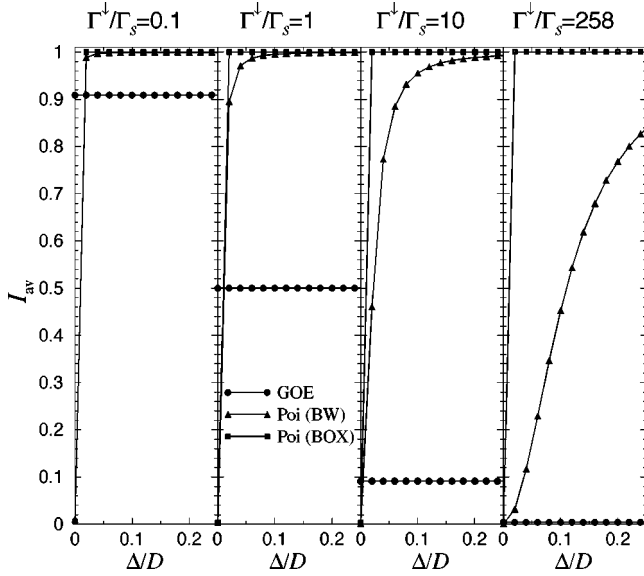


FIG. 1. Decay intensity  $I_{av}$  vs  $\Delta/D$ . The circles were calculated using Eq. (5), the triangles by substituting Eq. (6) into Eq. (8) and the squares using Eq. (9). We set  $\Gamma_N/D=3 \times 10^{-4}$  and  $\Gamma_S/D=6 \times 10^{-6}$  which are the relevant values for  $^{194}\text{Hg}$ -1 at spin  $12 \hbar$  [9].

In Ref. [7] the effect of the chaoticity of the ND states on  $I_{av}$  was investigated by varying the strength of the residual interaction of the  $|Q\rangle$  and their interaction with  $|d\rangle$ , both being assumed to be proportional to a parameter  $\lambda$  (the chaoticity parameter) which may be varied continuously in the range  $0 < \lambda \leq 1$ . The limiting value  $\lambda=0$  results in the  $|Q\rangle$  having Poisson statistics (regularity) while  $\lambda=1$  results in their having GOE statistics (chaos). The value of  $\lambda$  determines the shape of  $S_d(E)$  [and  $R_d(E)$ ] which is precisely the strength function that was investigated as a function of  $\lambda$  in Ref. [10]. In Ref. [8] it was pointed out that the calculations of Ref. [7] were restricted to  $E_d=E_0$ . We now study the Poisson limit of *model B* of Ref. [7] for  $E_d \neq E_0$ .

As  $\lambda \rightarrow 0$  and  $\Gamma_N \rightarrow 0$ ,  $S_d(E) \rightarrow \delta(E-E_d)$ . For nonzero  $\lambda$ ,  $S_d(E)$  broadens with increasing  $\lambda$  until when  $\lambda=1$  it takes a form well approximated by [10]

$$S_d^{\text{GOE}}(E) = \begin{cases} 1, & |E-E_d| \leq L/2, \\ 0, & |E-E_d| > L/2. \end{cases} \quad (4)$$

Inserting Eq. (4) into Eq. (1) we find that [11]

$$I_{av}^{\text{GOE}} = (1 + \Gamma^\downarrow/\Gamma_S)^{-1}, \quad (5)$$

as long as  $\Gamma_S + \Gamma^\downarrow \ll L$ .

Instead of studying the interpolation between the limits  $\lambda=0$  and  $\lambda=1$  by numerically diagonalizing random matrices and performing ensemble averages as was done in Refs. [7,10], we restrict ourselves to the limiting case  $\lambda=0$  and use two representations of  $\delta(E-E_d)$  broadened by  $\Gamma_N$ : the Breit-Wigner function,

$$S_d^{\text{BW}}(E) = \frac{1}{2\pi} \frac{\Gamma_N}{(E-E_d)^2 + \Gamma_N^2/4}, \quad (6)$$

and the box function,

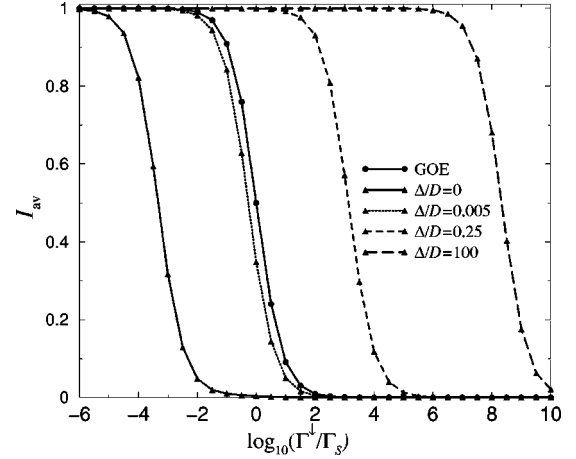


FIG. 2. Decay intensity  $I_{av}$  vs  $\log_{10}(\Gamma^\downarrow/\Gamma_S)$ . The meaning of the symbols and the values of  $\Gamma_N/D$  and  $\Gamma_S/D$  are the same as in Fig. 1.

$$S_d^{\text{BOX}}(E) = \begin{cases} 2/(\pi\Gamma_N), & |E-E_d| \leq \pi\Gamma_N/2, \\ 0, & |E-E_d| > \pi\Gamma_N/2. \end{cases} \quad (7)$$

Equation (1) for  $I_{av}$  depends on four parameters:  $\Gamma_S$ ,  $|V_{0d}|^2$ ,  $\Gamma_N$  and the distance in energy separating  $|d\rangle$  from  $|0\rangle$ ,  $\Delta=E_d-E_0$ . It is useful to introduce a spreading width defined by  $\Gamma^\downarrow=2\pi|V_{0d}|^2/D$ . Upon making the change of integration variable  $x=(E-E_0)/D$ , Eq. (1) takes the form [we set the energy shift  $R_d(E)=0$  as doing so does not modify our conclusions]

$$I_{av} = \frac{\Gamma_S/D}{2\pi} \int_{-\infty}^{\infty} dx \frac{1}{x^2 + (\Gamma_S/D)^2 \left[ 1 + \frac{\Gamma^\downarrow}{\Gamma_S} S_d(Dx+E_0) \right]^2 / 4}. \quad (8)$$

Inserting Eq. (7) for  $S_d$  into Eq. (8) we obtain

$$I_{av}^{\text{BOX}} = 1 + \frac{1}{\pi} \left[ \arctan \theta_- - \arctan \theta_+ + \frac{1}{1 + \frac{2\Gamma^\downarrow/\Gamma_S}{\pi\Gamma_N/D}} \times \{ \arctan \phi_+ - \arctan \phi_- \} \right], \quad (9)$$

where

$$\theta_{\pm} = \frac{1}{\Gamma_S/D} [2\Delta/D \pm \pi\Gamma_N/D] \quad (10)$$

and

$$\phi_{\pm} = \frac{\theta_{\pm}}{1 + \frac{2\Gamma^\downarrow/\Gamma_S}{\pi\Gamma_N/D}}. \quad (11)$$

From Eqs. (9)–(11) it is seen that as  $\lambda \rightarrow 0$ ,  $I_{av}$  is a function of four dimensionless variables:  $\Gamma^\downarrow/\Gamma_S$ ,  $\Gamma_N/D$ ,  $\Gamma_S/D$ , and

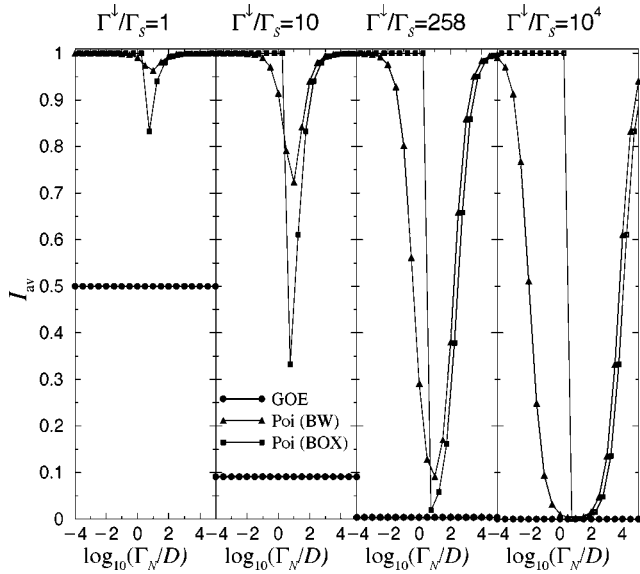


FIG. 3. Decay intensity  $I_{av}$  vs  $\log_{10}(\Gamma_N/D)$  for  $\Delta/D=0.25$ . The meaning of the symbols and the value of  $\Gamma_S/D$  are the same as in Fig. 1.

$\Delta/D$ . Figures 1–4 show  $I_{av}$  vs  $\Delta/D$ ,  $\Gamma^\perp/\Gamma_S$  and  $\Gamma_N/D$  and  $\Gamma_S/D$ , respectively. For the Poisson limit a significant dependence of  $I_{av}$  on all four parameters is observed. In all the graphs  $D=16.3$  eV,  $\Gamma_N=4.8 \times 10^{-3}$  eV and  $\Gamma_S=97 \times 10^{-6}$  eV which are the values for  $^{194}\text{Hg}$ -1 at spin  $12 \hbar$  [9]. The triangles and squares represent the Breit-Wigner and box function representations of the Poisson limit, respectively, while the circles represent the GOE limit. The decay-out is enhanced by increasing the degree of chaos if the triangles or squares are above the circles and it is hindered if the triangles or squares are below the circles.

The authors of Ref. [9] obtained a spreading width of  $\Gamma^\perp=0.025$  eV ( $\Gamma^\perp/\Gamma_S=258$ ) from an experimental value for the total intraband decay intensity at spin  $12 \hbar$  equal to 0.58, using the theory of Ref. [12]. They assume that the fluctuation contribution dominates ( $J_{av}^{\text{GOE}}=1/259$ ). It is clear from Fig. 1 that the extraction of  $\Gamma^\perp$  from experimental data using the results for the Poisson limit of the present paper would be extremely sensitive to  $\Delta$ . The energy difference  $\Delta$  is an additional unknown parameter.

From Figs. 1 and 2 we see that increasing the chaoticity,  $\lambda$ , from 0 to 1 only hinders the decay out when  $\Delta \sim 0$  as was observed by Åberg [8]. However, even though a chaos enhancement is obtained for  $\Delta$  sufficiently large, it is more

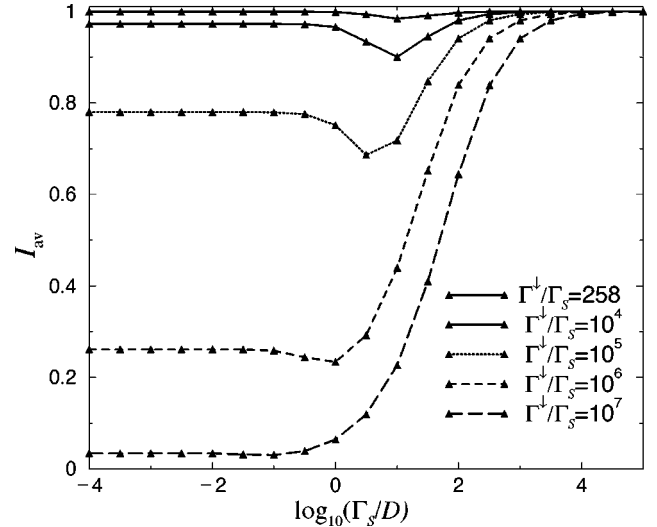


FIG. 4. Decay intensity  $I_{av}$  vs  $\log_{10}(\Gamma_S/D)$ . The meaning of the symbols and the value of  $\Gamma_N/D$  are the same as in Fig. 1.

convincing to explain the decay-out by an increase in  $\Gamma^\perp/\Gamma_S$ , than by an increase in  $\lambda$ , for the following reasons: First, we see from Figs. 1–3 that increasing  $\lambda$  from 0 to 1 cannot exhaust all of the intraband decay intensity unless  $\Gamma^\perp/\Gamma_S \rightarrow \infty$ . Indeed, the extent to which the transition from ordered to chaotic states may exhaust the intraband decay intensity is determined solely by  $\Gamma^\perp/\Gamma_S$  [ $I_{av}^{\text{GOE}}=(1+\Gamma^\perp/\Gamma_S)^{-1}$ ]. For example, when  $\Gamma^\perp/\Gamma_S=0.01$ , a transition from ordered to chaotic states will reduce (if  $\Delta \gg 0$ )  $I_{av}$  from 1 to 0.99—a rather small effect. Further, when  $\Gamma^\perp/\Gamma_S=0$  it is impossible for an increase of chaos to trigger the decay-out since  $I_{av}=1$  for values of  $\lambda$  [see Eq. (8)]; second, a chaos-order transition is not necessary to trigger the decay-out since  $I_{av} \rightarrow 0$  as  $\Gamma^\perp/\Gamma_S \rightarrow \infty$  whatever the values of  $\lambda$  and  $\Delta$  as long as  $\Gamma_N$  has a nonzero value [see Eq. (8)]. It may be seen from Fig. 2 that this is the case even for  $\lambda=0$  [when  $S_d(E)$  is described by the Breit-Wigner function].

It is true to say that  $\lambda$  modifies the spin at which the decay-out occurs as can be seen from Fig. 2 (see also Fig. 3 of Ref. [7]). However the arguments of the preceding paragraph convince us that the spin dependence of  $\Gamma^\perp/\Gamma_S$  is of more importance. Since  $\Gamma^\perp/\Gamma_S$  is determined by the deformation barrier these arguments reinforce our belief in the conclusion of Ref. [7] that the attenuation of the intraband intensity with decreasing spin is mostly due to the spin dependence of the barrier.

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- [15] Equation (1) is in fact the background contribution to the average decay intensity. The fluctuation contribution should be added to  $I_{av}$  [12–14]. Further, it appears possible that the variance of the decay intensity contains useful information about the decay-out mechanism [12–14].