

The Mathematics of the Curves on the Wall of the Colégio Arquidiocesano¹ and its Mathematical Models: a Case for Ethnomodeling

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Abstract

In this article the authors share the results from conversation during a morning walk with students along a street in Ouro Preto. The simple observation that a phenomena one might see everyday may possess mathematical potential became an interesting debate topic that turned this observation into an exploration of the mathematics inherent in common architectural details. There are many similar conversations and situations in any city that can be used to encourage conversation, develop models and explore the relationships between mathematical ideas, procedures, and practices. The authors studied the mathematical models derived from curves along the wall and sought to verify if they were related to exponential, parabolic, or catenary curves. In order to make the necessary arguments to support these conjectures, models were elaborated and analyzed, and discussed through a methodological procedure in which some curves were randomly selected from the wall of the school. After examining the data they came to the conclusion that the curves on the wall approximate that of a catenary curve.

Keywords: Catenary, Mathematical Modeling, Ethnomodeling.

1 Introduction

As a visiting professor and now a professor at the Universidade Federal de Ouro Preto² professor Orey began a project which has come to be known as the *Trilha de Matemática de Ouro Preto*³ (Ouro Preto Math Trail). The project began in 2005-2006 during which time the Prefeitura (City Hall) of Ouro Preto developed a program called *O Museu Aberto (The Open Museum)* that was designed to encourage people to take pride in their city. In this program, the city developed historical routes that encouraged people to walk in different neighborhoods of the town and to read the informative plaques placed on homes and structures of important buildings, homes, and historical points of interest throughout the city. Figure 1 shows the view of Ouro Preto, in the state of Minas Gerais, in Brazil.

¹Colégio Arquidiocesano is located on the Rua Alvarenga, which is an important street, in Ouro Preto, in the state of Minas Gerais, in Brazil.

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³<http://sites.google.com/site/trilhadeouropreto/>



Figure 1: Ouro Preto

Source: Photographed by Daniel Clark Orey

A course on mathematical modeling was taught later in that school year by Prof. Orey who had been documenting numerous opportunities for modeling research, and collaborating with numerous university students and with the Ouro Preto Municipal Schools. One example, from the *Trilha* was that we first observed was an interesting pattern in the wall of the *Colégio Arquidiocesano* along *Rua Alvarenga*. Figure 2 shows the *Colégio Arquidiocesano*. The wall can be seen at bottom left of the figure.



Figure 2: The Colégio Arquidiocesano

Source: http://www.arquidiocesano.com/?page_id=14

The observed pattern soon formed part of a debate, then a passion of the authors of this investigation. Figure 3 shows the architectural pattern on the wall of the *Colégio Arquidiocesano*⁴ on the *Rua Alvarenga* in Ouro Preto.

⁴The *Colégio Arquidiocesano* of Ouro Preto was founded by Dom Helvécio Gomes de Oliveira, in 1927, functioning as a school in the old Palace of the Bishops in Mariana, Minas Gerais in Brasil. Figure 3 shows the current collection of buildings that was constructed in 1933. In 1934, the *Colégio Arquidiocesano* began its activities in Ouro Preto.



Figure 3: One section of the architectural pattern on the wall of the Colégio Arquidiocesano
Source: Photographed by Daniel Clark Orey

In this regard, we would like to share with the readers, the background, and elements of our discussions, data, and final observations about a simple architectural pattern that may encourage the readers to undertake similar explorations in your own community.

2 Mathematical Modeling and its Relation to the Community Context

Mathematics is a dynamic, changing, and active system of knowledge. To learn to use mathematics, therefore, does not mean that we passively receive or memorize ready-made concepts in prepackaged forms. According to Freire (1970), this educational approach is called the *banking model* of education.

The true learning of mathematics is like that of learning to be an artist, or athlete, as in learning how to play soccer, or in the mastering of a video game. In so doing, we believe that any real learning of mathematics is intimately related to gender, sexual orientation, age, work, reality, language, spirituality, and the unique thoughts and emotions all of which come to define culture.

It is the construction of mathematical concepts that should incorporate the reality and context of individuals (Rosa & Orey, 2007). This should begin by placing new situations and problems in front of learners for them to master within the context of their own unique cultural and experiential reality. It is only on this basis that new mathematics concepts are developed for the *construction* or *deconstruction* of an individual's link to the larger mathematical universe.

Beginning with a learners' experience it is as logical as it is pedagogically sound, and it is with a sight towards the understanding, comprehension, and resolution of real problems that is often at odds with how mathematics is taught and learned in schools. We agree with Orey and Rosa (2004) that learners of mathematics must be given experiences to enable them to learn how to:

1. Break a problem situation into manageable parts.
2. Create their own hypothesis.
3. Test the hypothesis they developed.
4. Check and correct their hypothesis.
5. Make transference and generalizations to their own reality.

Traditionally, academic mathematics has become symbolic of the afore mentioned *banking model* (Freire, 1998) in which students face subject matter as passive learners and come to mathematics classes with a marked level of anxiety and distaste for the subject. A quick search on social network sites such as *Facebook* uncovers numerous groups dedicated to anti-mathematics sentiments⁵. On the other hand, there is a marked need to feel empowered, to participate in making informed decisions that are often mathematically based, and the need to direct one's own learning (Rosa, 2010). Yet, this is virtually ignored by current attempts at school, curricular, pedagogical, and mathematical reform.

For instance, computer gaming enamors students in many countries. One can safely say that it is a worldwide phenomenon, with millions of gamers playing against each other online and across

⁵<http://sciencercay.com/mathematics/why-do-people-hate-math/>

political, linguistic, and cultural borders (Krotoski, 2012; Powers, 2011). Whether educators like it or not, the reality is that, many young people are comfortable using this technology, and through an intense interest in the use of smart media and computer games, their anxiety level is greatly diminished and a sense of accomplishment can be strengthened (Ahl, 1981; Betz, 1995; Fengfeng, 2006).

The use of technology allows students to be creative and utilize alternative forms of intelligences. It also allows them to build on both self-esteem and collegiality (Johnson & Johnson, 1996). Thus, mathematics activities should vary in the use of a variety of contexts, tools and technologies (CDE, 1992; MEC, 1996; NCTM, 1989) to enable opportunities for:

- Appropriate project work.
- Group and individual assignments.
- Discussion between teacher and students and among students.
- Practice on mathematical methods.
- Exposition and modeling by the teacher.

An effective mathematics learning environment encourages a deep understanding and allows users to debate and interpret their own ideas and findings (Barbosa, 2001; Rosa, 2005). Previous experiences that students possess take-on new forms of knowledge by evolving new and increasingly complex contexts, ideas and products as they learn to solve problems that exist in their own communities (Rosa & Orey, 2007). This often stands as a creation incorporating the historical-cultural nature of mathematical concepts, by assisting educators and students to reflect in context on the mathematical processes that they use (D'Ambrosio, 1998; Gutstein, 2006). As they gain more confidence in developing new ideas for themselves, they show the presence of mathematics in their daily lives through dialogue and conversation.

Using this paradigm, one learns to explore and explain alternative ways to work within a mathematically based reality which we refer to as *transformative action*. Transformative actions look to reduce the degree of complexity, through the choice of a system⁶ (Freire, 1970). In this newly isolated system, the representations that emerge are related to this reality enable the elaboration of ongoing strategies that *explore, explain, and increase* comprehension of the transformation being described.

The study of the actions in this system allows us the opportunity to reflect on the possibilities inherent within it and for it to become an object of critical analysis of the work done by the learners themselves (Bassanezi, 2002; D'Ambrosio, 1993). This is not unlike groups of gamers playing an online game together. We define the process by which we *consider, analyze, and make* ongoing and *critical* reflections on a system that is taken from our reality as ethnomodeling (D'Ambrosio, 2002; Rosa & Orey, 2010).

These applications are inserted during the modeling process and are made through the system's reality in which they instituted the model. This of course affects ongoing and future analyses and reflections because the ongoing system is an integral part of the considered reality; that is, it is a set of items including the inter-relatedness between these essential components (D'Ambrosio, 1993).

In acquiring knowledge by using ethnomodeling practices, we learn how to work with new approaches and new ideas by which we construct our own representations of the actions, systems and/or problems under study (Bassanezi, 2002, Orey & Rosa, 2012). Established relations that have been constructed between the system and its representations make the research and subsequent findings valid; this is done through triangulation, dialogue and analyses of the various models that often end-up guiding this pedagogy. Through the incorporation of significant and motivating pedagogical activities such as the one we share below, the learning process itself becomes a mathematical reality.

Bassanezi (2002) stated that the measure by which this is incorporated into both knowledge and concepts that are essential for future performance in society are true marks of success. However,

⁶ A system is a part of reality considered integrally (Rosa, 2000). According to D'Ambrosio (1993), a system is a set of items taken from the reality, components, and the interrelationships between these components.

this incorporation is not given through simple, passive or blind adhesion to tests, rote learning, and passive instructional materials.

Again, we encourage readers to think of computer gaming as a metaphor here. In so doing, modeling has as its main objective the development of reflections and a real participation by the learners themselves. In so doing they will be better able to trace parallels between their experiences and reality. This educational approach includes thinking in a critical and historical nature because it uses representations that must underlie the modeling process (Bassanezi, 2002; D'Ambrosio, 1993; Rosa & Orey, 2008).

Involvement in modeling encourages an inner transformation that manifests itself as action when questions are both formulated and answered (D'Ambrosio, 1993). The engaged *hunger to know* is a manifestation of this *transformational* process when together the teachers and students engage an inquiry process and demonstrate connections to required academic content that is not just subject, content or assessment driven (Rosa & Orey, 2007). And once again, this is precisely what is observed when people are engaged in sports or computer gaming (Krotoski, 2012).

Beginning with problems found in the day to day lives in the community, activity such as this allows educators to develop skills as facilitators and to create conditions for increased involvement making mathematics accessible for all (MEC, 1996; NCTM, 1989). Ethnomodeling empowers both teachers and students to create a realistic climate where by errors that naturally occur in context are part of a teaching-learning process (Rosa, 2010). In this way, it becomes significant, especially when good models are elaborated and demonstrated (Bassanezi, 2002), and this happens when students learn at the same time as they apply their models. Ethnomodeling produces, indeed develops, further models that are equally interesting and useful. This process enables learners to learn how to engage in a process of continual modification, to alter their reflections, to develop mature arguments, to create ongoing analysis and to develop solid conclusions regarding their models (Biembengut, 1999) and allows them to experience the scientific method within the context of learning mathematics.

After the questioning activities are initiated, models are further elaborated by students and then demonstrated (Rosa, 2000). In this way, students learn to apply modeling as an educational methodology and to experience an ethnomathematical perspective as a practical pedagogy. This process produces and develops models that enable further development and modifications through collaborative decision making. After further reflections, discussion, analysis has taken place, elaborated models are presented, and findings shared with peers and colleagues (Rosa & Orey, 2004). When learners are actively involved in the construction of their own learning, the waste of their enthusiasm and innate energy does not occur (Rosa & Orey, 2003).

Mathematical modeling is the study of problems or situations for understanding, simplification, and resolution with sights for a possible forecast or modification of the studied object (Bassanezi, 2002). Ethnomodeling as a learning strategy values the students' own knowledge and stimulates academic performance in learners; and as pedagogical action, discloses the potential of mathematics by using models as efficient methodological tools useful in science and technology-based environments (Orey, 2000).

This ongoing and organic process whereby modelers both create and redefine is important for students as they develop deeper mathematical understanding in the context of phenomena from their own reality. A powerful and valid introduction to mathematical modeling is to expose students to a diversity of realistic and engaging problems taken from their own context (D'Ambrosio, 1993). These include mathematical interpretations of problems that in turn are representations of the systems under study. Therefore, when we analyze a given situation for its mathematical perspective, the teaching-learning process itself stimulates thought, and is not just the mere memorization of basic facts and algorithms taken from traditional texts or materials (Burak, 1992).

We define an ethnomodel as a body of symbols and mathematical relationships representing a studied object taken from a unique cultural context. This particular form of modeling applies a system of equations or inequalities, algebraic expressions, differentials, and integrals obtained through the establishment of a relationship between considered essential variables of analyzed phenomena (Bassanezi, 2002) that translates diverse mathematical content into the modern language of mathematics. In other words, ethnomodels are almost always describing and translating systems of doing mathematics into algebraic or differential equations, obtained through the establishment of

relations between variables that are considered essential in an analyzed phenomena. However, a model is not just a set of quantitative variables that make qualitative representations of the system being analyzed.

2.1 Inaccuracy of Models

What happens when models are not accurate? When this becomes evident, the exploration of the unique details of the model and the examination of new or emerging hypotheses is essential (Rosa, 2005). Checking the preciseness of the calculations, and making necessary adjustments to the model (Caldeira, 2004) is also essential in the scientific/modeling method. When this is done, new forecasts are developed that validates the hypotheses.

Considering these relationships, one might conclude that learning to model consists of applying techniques necessary to solve connected mathematical problems in the systems under study. However, it is not possible to explain, to know, to understand, or to carry-out the reality outside of a truly holistic⁷ context because the results can never be anything but than partial or incomplete visions of reality unless they can be seen as connected holistically (D'Ambrosio, 1993). And this is why it is called a model, it is a partial understandign of a greater reality. At the same time, it is important not to confuse mathematical modeling with a mere group of formulas, theories or techniques that resolve mathematical models because formulas which are memorized and later forgotten as disconnected irrelevant pieces of data (Rosa & Orey, 2008). During this ongoing resolution process, participants develop a continual relationship between the model and the *modeler*⁸.

With a growing confidence and experience, modelers develop a general idea related to the type of model that they seek to elaborate. This idea can also take on aspects of a hypothesis. At this stage, it is normal that students do not possess a clear or detailed notion about what they hope to find. Modelers define the characteristics of their model as they go along, so that they can have a general idea as to the objectives that they are seeking to attain. This is a process of constant experimentation, investigation, and (re)discovery (Biembengut, 1999).

Both educators and teachers must help students to work with significant and contextualized mathematics (Rosa, 2000), which consists of two processes: the creation of abstract mathematics (academic or pure mathematics) and the application of mathematics (mathematical modeling) to solve questions outside the field of mathematics (interdisciplinary) to connect ideas within mathematics and other intellectual activity (mathematical models). In other words, students need opportunities to work on authentic situations and real-world problems in order to connect their previous knowledge with the academic knowledge acquired in mathematics classrooms.

3 Ethnomodeling: The Cultural Aspects of Mathematical Modeling

Ethnomodeling is the process of elaboration of problems and questions that grow from real situations forming images or a sense of an idealized version of *mathema*, which is considered as the natural, cultural, political, economic, and social environments in which we use to explain and understand daily phenomena. In agreement with D'Ambrosio (1990), this perspective allows for a critical analysis of the generation and production of knowledge (creativity), and forms an intellectual process for the production, the social mechanisms of the institutionalization of knowledge (academics), and its transmission (education). This process is what we refer to as *ethnomodeling* (D'Ambrosio, 2002; Rosa e Orey, 2010).

By analyzing reality as a whole, this allows those who are engaged in the modeling process to study whole systems of reality in which there is an equal effort made by them to create an

⁷ A holistic context consists essentially of a critical analysis of the generation (creativity) of knowledge, and the intellectual process of its production. The focus on history analyzes the social mechanism and institutionalization of knowledge (academics), and its transmission (D'Ambrosio, 1998).

⁸ A modeler is a person (student, teacher, educator, engineer, biologist, etc) who takes out the particular system from its reality for study (Rosa, 2000). In so doing, the modeler uses a mathematical modeling methodology to develop a mathematical model strategy to find a solution for real-world problem. The modeler transforms an idealized form of a real-world situation into mathematical and concrete terms.

understanding of all components as well as their interrelationships (D'Ambrosio, 1993; Bassanezi, 2002). The use of ethnomodeling values previous knowledge and traditions by developing the student capacity to assess and translate processes that elaborate models in their diverse contexts and applications and by having started with the social contexts, realities and interests of students and not by enforcing a set of external values and curricula without context or meaning for learners. We call this process ethnomodeling because it involves the mathematical practices present in diverse situations in the daily lives of members of these diverse groups" (Bassanezi, 2002, p. 208). In this regard, ethnomodeling is much more than the standard transference of knowledge. Here, teaching becomes an activity that introduces the creation of knowledge (Freire, 1998), it is transformative. This approach in mathematics education is the antithesis of turning students into containers to be filled with information (Freire, 1970).

Ethnomodeling uses mathematics as a language for understanding, simplification and resolution of real world problems and activities. Data gleaned from these studies are used to make forecasts and modifications pertaining to the objects initially studied. One of the traditional definitions of a mathematical model is a body of symbols and mathematical relationships that represent the studied object, which is composed by a system of equations or inequalities, algebraic expressions, differentials, and integrals that are obtained through the establishment of a relationship between considered essential variables of analyzed phenomena (Bassanezzi, 2002). In other words, it is the systematic study of algorithmic processes, theory, analysis, design, efficiency, implementation, and application, which describes and transforms information. This definition of the Western mathematical modeling includes all data structures, which is a part of both *theory* and *design*; algorithms that concerned with an analysis, efficiency; mechanical and linguistic realizations. It is equally concerned with implementation; and applications that naturally applies the mathematical ideas and concepts to solve problems.

The importance of a non-traditional view of mathematics is emphasized with the emergence of new types of problems related to artificial intelligence. A characteristic of these new problems is that they cannot be solved using syllogistic, that is, classical Aristotelian logic, but need multivalued logic, often called *fuzzy logic*, which is the logic that underlies inexact or approximate reasoning (Zadeh, 1984). According to Ascher and Ascher (1986), multivalued logic is used in attempts to formalize human-like culturally bound processes. From this perspective, Hindu, Chinese and Japanese cultures have contributed to the development of fuzzy logic just as much as traditional Western science because, in these cultures, there is a greater acceptance of a truth-value that is neither perfect truth nor perfect falsehood (Zadeh, 1984).

The purpose of the above discussion has been to introduce the readers to how the authors have engaged in the process of ethnomodeling to study the wall along the Colégio Arquidiocesano in Ouro Preto. Next we will share brief exploration of our work related to understanding what are parabolas and catenaries.

4 A Brief History of Parabolas and Catenaries

Possibly, the first known mathematicians who discussed the connections between parabolas and catenaries had assumed that a catenary was a parabola. In the 17th century, inspired by the success of his work with conic sections, Galileo Galilei incorrectly believed that the hanging of a heavy rope would hang in the shape of a parabola (Lockwood, 1961). Galileo was the first to investigate the catenary, which he mistook for a parabola. It was the German mathematician Jungius who successfully demonstrated the incorrectness of Galileo's earlier assumption (Lockwood, 1961).

The first recorded term *catenary* was first used by Huygens in a letter to Leibniz in 1690 to describe the curve that is defined by the shape of a flexible hanging wire or chain that is supported at its ends and acted upon by a uniform gravitational. The equation of the curve was only obtained by Leibniz, Huygens, and Johann Bernoulli in 1691 (Lockwood, 1961). In addition to the work of these mathematicians, the Swiss mathematician Bernoulli and the German mathematician Leibniz also contributed to the complete description of the equation of the catenary⁹ and described some of its properties (Yates, 1974). David Gregory, an Oxford professor, wrote a comprehensive treatise on the

⁹ <http://concise.britannica.com/ebc/article-9047669/Gottfried-Leibniz>

catenary in 1697 (Lockwood, 1961), and it was Euler who proved in 1744 that the catenary is the curve which, when rotated about the x -axis, gives the surface of minimum surface area, or the *catenoid*, for the given bounding circle.

4.1 Similarities and Differences between Parabolas and Catenaries

The parabola and the catenary are two different curves that look extremely similar. Both are symmetrical and have a cup shape, going up infinitely on either side of a minimum value. In figure 4, the blue graph represents the parabola and the red graph represents the catenary. They were graphed together, so we can see the similarities and differences between these two graphs.

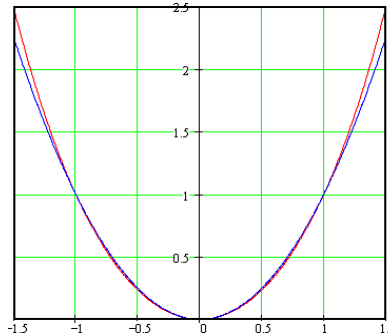


Figure 4: The graph of a parabola (blue) and the graph of a catenary (red).

The mathematical equation $y = \frac{\cosh(x)-1}{\cosh(1)-1}$ is the hyperbolic cosine function of the catenary.

When we move the vertex from $(0, 1)$ down to the origin, we can make it agree with the parabola when $x = 1$. We would like to make clear that the catenary is slightly more *flat* at the bottom and that it rises faster than the parabola for large values of x . In fact, if we graph the parabola and the catenary for a larger domain, the catenary would be far higher than the parabola.

It is important to note that the hyperbolic cosine function may also be used to describe the shape of the catenary curve formed by a high-voltage line suspended between two towers. However, if the two fixed endpoints are at the same height, the catenary that is joined by these two points may look like a parabola. While parabolas have the shape of the curve $y = x^2$, catenaries have the shape of the graph of the hyperbolic cosine function $y = \cosh(x)$.

In this context, parabolas and catenaries are mathematically distinct types of curves. However, they have some similar mathematical properties. For example, if the concave of both curves opens upward, they:

- Have a single low point. In this case, the lowest point of the parabola is called its *vertex*, and the lowest point of the catenary is called its *apex*.
- Have a vertical line of symmetry.
- Appear to be continuous and differentiable throughout the shape of the curve.
- Have slopes that are steeper as they move away from the lowest point, however, the curves never become a vertical line.
- May be generated by hanging a flexible cable, wire, or chain between two fixed points.

On the other hand, the difference between the two curves is related to different ways that the weight is distributed along the length of the each cable, chain, string, or wire. For example, there are two cases to be considered:

1. If weight is distributed evenly along the length of the chain, string, or wire, then the result is a catenary. In other words, the only weight attached to the chain is the weight of the chain itself.
2. If weight is distributed evenly along a horizontal line, then the result is a parabola.

As for why the St. Louis Gateway Arch, or a hanging cable, takes the shape of a catenary, while the cables on a suspension bridge form a parabola that is just a result of the physics of each

situation. By applying calculus, we are able to verify that the catenary is the solution to a differential equation that describes a shape that directs the force of its own weight along its own curve, so that, if hanging, it is pulled into that shape, and, if standing upright, it can support itself.

It is important to emphasize that, in mathematics; the catenary is the shape of a curve described by an ideal string that is suspended by its two endpoints. In this case, the ideal means that the string is perfectly flexible, that is, the curve is inextensible, with no thickness, with uniform density, and is subjected to only the influence of gravity. This shape imparts great strength to structures when built in the shape of a catenary.

For example, figure 5 shows the Gateway Arch, in Saint Louis, Missouri, which has the shape of an inverted catenary, with 630 feet wide at the base and 630 feet tall. The formula of this arch is given by $y = -127.7 \cosh\left(\frac{x}{127.7}\right) = 757.7$.



Figure 5: The Saint Louis Gateway Arch
Source: Photographed by Rick Dikeman

On the other hand, the parabola does not have the same property, but is the solution of other important equations that describe other situations. For example, in nature, approximations of parabolas are found in any number of diverse situations. In the history of physics there is the trajectory of a particle or body in motion under the influence of a uniform gravitational field without air resistance such as the parabolic trajectory of baseball and projectiles.

Parabolic shapes are also found in several physical situations such as parabolic reflectors commonly observed in microwave or satellite dish antennas. The mathematical properties of parabolas make them excellent models for physical objects in which a focusing component is essential. It can be shown that parallel lines drawn on the inside of any parabola are reflected from the curve of the parabola to its focus. Thus, many telescopes and satellite television receivers are designed using parabolic reflection properties. Figure 6 shows the antenna of a radio telescope, which resembles a parabola.



Figure 6: Antenna of a Radio Telescope

Source: <http://www.carondelet.pvt.k12.ca.us/Family/Math/03210/page3.htm>

Parabolas also model the motion of a body in free fall towards the surface of the Earth and are used in the design of bridges and other structures involving arches. So after looking at the designs on

the wall, we decided to compare them with what we knew about parabolic or catenary functions and how they related to suspension bridges.

4.2 The Specific Case of Suspension Bridges

In light of the facts discussed previously, the following question was formulated: *Do cables of suspension bridges have a catenary shape?* Figure 7 shows the construction proceeds on the Brooklyn Bridge, in 1881, in New York.



Figure 7: Construction of the Brooklyn Bridge
Source: Museum of New York¹⁰

The answer to the above question is *no*. In this specific case, it is interesting to note that when suspension bridges are constructed, before the suspension cables are tied to the deck below them, they initially have a hyperbolic cosine function shape, that is, the shape of a catenary. This happens when the structure of the bridge is being built and when the main cables are attached to the towers and then, the cables are attached to the deck with hangers. In so doing, the cable of a suspension bridge is under tension from holding up the bridge. The cable is also under the influence of a uniform load, that is, the deck of the bridge, which deforms the cable into a parabola. This deformation happens because the weight of the deck is equally distributed along the curve. In this regard, it is possible to conclude that the catenary curves under its own weight while the parabola curves under both its own weight and by holding up of the weight of the deck.

5 Methodology: Modeling the Wall by Searching for Mathematical Models

It must be noted here, that we were interested in verifying if we could prove that the patterns found along the wall of the school were a series of exponential curves. That is, was the wall representative of a *parabola* or a series of *catenaries*?

What happened in Ouro Preto was something altogether surprising, and in the end, the final results, important as they are, were eclipsed by the actual opportunity we had to discuss and debate aspects related to exponential curves, parabolas, and catenaries. Not unlike archeologists or ongoing restoration efforts in Ouro Preto, as our work progressed and the group had discussions with passersby as data was gathered, and as we worked on the models, and discussed mathematical concepts, and patterns that many had never taken much notice of along Rua Alvarenga. What we have attempted to describe here in this article was both the process and discussion about mathematical processes by which we came to discuss and determine several mathematical models for the shapes on the wall of the Colégio Arquidiocesano.

In so doing, by observing the architectural drawing of the façade of the Colégio Arquidiocesano we were able to determine ways to relate the functions of these three curves, exponential, parabolas, and catenaries to the patterns found on the wall of the school. Figure 8 shows the patterns of the shape on the wall of the Colégio Arquidiocesano.

¹⁰ <http://www.endex.com/gf/buildings/bbridge/bbridgenews/AmHist/image12x.jpg>



Figure 8: The Pattern on the Wall
Source: Photographed by Daniel Clark Orey

So, in getting to the object of our study, it was interesting to talk about if the curves had an exponential, parabolic or catenary shape. In order to have the necessary arguments to answer this conjecture, some mathematical models were elaborated and analyzed, and discussed.

5.1 Methodological Procedures

Initially, some curves were randomly selected on the wall of the school and an x-y coordinate system was constructed by using some parts of the wall as a plane. Strings were used to determine the x and y axes. After a brief discussion between Prof. Orey and Prof. Rosa, it was decided that the origin of the x-y coordinate system would be placed on the lowest point of one of the curves. At that point, there was no certainty if the lowest point was the *vertex* or the *apex* of the curve. Figure 9 shows the curve on part of the wall of Colégio Arquidiocesano with the strings that represent the x and y axes.



Figure 9: Curve on the wall of Colégio Arquidiocesano with the x and y axes
Source: Photographed by Daniel Clark Orey

After that procedure, some points were selected and placed on the curve. Table 1 shows the coordinates of the points that were placed on the selected curve on the wall.

x (in meters)	y (in meters)	Points
-0.925	0.23	A (-0.925; 0.23)
-0.745	0.15	B (-0.745; 0.15)
0	0	C (0; 0)
0.7	0.125	D (0.7; 0.125)
0.91	0.2	E (0.91; 0.2)

Table 1: Points placed on the selected curve

The points placed on the selected curves were placed in a table and these coordinates were calculated and values were determined for each point.

6 Results and Discussions

Some necessary adjustments were made to the points of the selected curves in order to have an accurate value of their coordinates. Below, we show the results and discussion of two mathematical models developed from data collected on curves of the wall of the school.

6.1 First Mathematical Model: The Adjustment of an Exponential Curve to a Catenary

First, we attempted to adjust the points for an exponential curve. By using the following collected data:

$\cosh(-0.925) \cong 1.4592$
$\cosh(-0.745) \cong 1.2906$
$\cosh(0) = 1$
$\cosh(0.7) \cong 1.2552$
$\cosh(0.91) \cong 1.4434$

Thus, we looked for an adjustment of the exponential curve that passes through the points A, B, C, D, and E in order to compare the exponential and the catenary curves. In this case, if g is the function in which the curve will be adjusted, then $g(0)$ must be equal to 1, that is, the distance D (in meters) from the x-axis to the x-axis, which was constructed initially, must be analyzed. Therefore, we determined the following results:

$\frac{0.4592 - 0.23}{1 - d_1} \Rightarrow d_1 \cong 0.5009$
$\frac{0.2906 - 0.15}{1 - d_2} \Rightarrow d_2 \cong 0.5162$
$\frac{0.2552 - 0.125}{1 - d_3} \Rightarrow d_3 \cong 0.4898$
$\frac{0.4434 - 0.2}{1 - d_4} \Rightarrow d_4 \cong 0.4511$

Therefore,

$$D = \frac{d_1 + d_2 + d_3 + d_4}{4} = 0.4895$$

In order to understand the graphs to be constructed, it is important to clarify the relationship between the coordinates of points A, B, C, D and new points A', B', C', D' e E' by using a simple *rule of three*¹¹, where C' (0; 1). However, the following observation is necessary here.

$$\begin{array}{l} 1 - 0.4895 \\ k - 0.7195(= 0.23 + 0.4895) \end{array} \Rightarrow k = 1.4699$$

Thus, A' = (-0.925; 1.4699). Where, | 1.4699 - 1.4592 | = 0.0107.

$$\begin{array}{l} 1 - 0.4895 \\ k - 0.6395(= 0.15 + 0.4895) \end{array} \Rightarrow k = 1.3064$$

Thus, B' = (-0.745; 1.3064). Where, | 1.3064 - 1.2906 | = 0.0158.

$$\begin{array}{l} 1 - 0.4895 \\ k - 0.6145(= 0.125 + 0.4895) \end{array} \Rightarrow k = 1.2554$$

Thus, D' = (0.7; 1.2554). Where, | 1.2554 - 1.2552 | = 0.0002.

$$\begin{array}{l} 1 - 0.4895 \\ k - 0.6895(= 0.2 + 0.4895) \end{array} \Rightarrow k = 1.4086$$

Thus, E' = (0.91; 1.4086). Where, | 1.4086 - 1.4434 | = 0.0348.

Figure 10 shows the graph of the adjustment of the exponential curve in relation to the catenary.

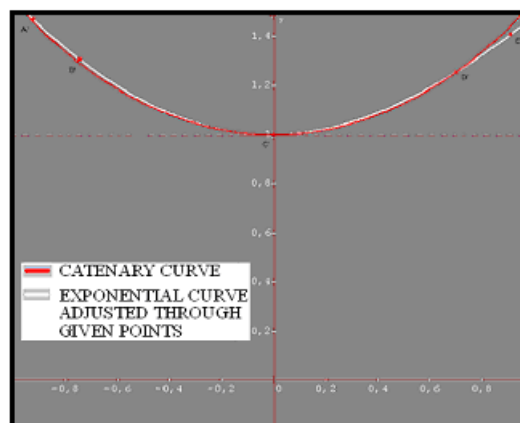


Figure 10: The adjustment of the exponential curve to the catenary

¹¹The Rule of Three was a shorthand version for a particular form of cross multiplication, often taught to students by rote. This rule was known to Indian mathematicians in the 6th century BCE and Chinese mathematicians prior to the 7th century CE, even though it was not used in Europe until the 17th century (Shen, Crossley, & Lun, 1999).

6.2 Second Mathematical Model: The Adjustment of a Quadratic Function to a Catenary

It is easily observed that the equation of the given graph was of the form of $y = rx^2 + sx + t$ and that there is no symmetry regarding to the curve in relation to the y-axis. In so doing, we adjust this curve to a quadratic function on the point $C' = (0; 1)$ by using the vertex of the original equation that was already observed in the definition of the parabola. In other words, the equation for the function has the form of $(x - 0)^2 = 2p(y - 1)$ or the equation may also have the form of $y = rx^2 + s$.

Therefore, we can observe that the equation $y = rx^2 + sx + t$ could not have the symmetry of the curve in relation to the y-axis or if the equation had the form of $y = rx^2 + s$. Therefore, the equation has the form of $y = rx^2 + sx + t$ and it would not have symmetry of the curve in relation to the y-axis. Then, if $y_1 = ax^2 + b$ and considering points A' and C' that lie on the graph of y_1 , it is possible to determine coefficients a and b . In this case, we have that:

i) $y_1(0) = 1 \Rightarrow b = 1$
ii) $y_1(-0.925) = 1.4699 \Rightarrow 0.855625a + 1 = 1.4699 \Rightarrow a = 0.5492$

Therefore, we have:

$$y_1 = 0.5492x^2 + 1$$

In this regard, for points B' , D' and E' , we determined that:

$y_1(-0.745) \cong 1.3048$.	Where, $ 1.3048 - 1.2906 = 0.0142$
$y_1(0.7) \cong 1.2691$.	Where, $ 1.2691 - 1.2552 = 0.0139$
$y_1(0.91) \cong 1.4548$.	Where, $ 1.4548 - 1.4434 = 0.0114$

However, it is necessary to observe that:

$y_1(-0.925) = 1.4699$	Where, $ 1.4699 - 1.4592 = 0.0107$
$y_1(0) = 1$	Where, $ 1 - 1 = 0$

Since $y_2 = ax^2 + b$, we can consider that points B' and C' lie on the graph of y_2 . In so doing, it is possible to determine coefficients a and b . Thus, we have that:

i) $y_2(0) = 1 \Rightarrow b = 1$
ii) $y_2(-0.745) = 1.3064 \Rightarrow 0.555025a + 1 = 1.3064 \Rightarrow a = 0.552$

Therefore, we have that

$$y_2 = 0.552x^2 + 1$$

From the previous formula, using points A' , D' and E' , we have that:

$y_2(-0.925) \cong 1.4723$	Where, $ 1.4723 - 1.4592 = 0.0131$
$y_2(0.7) \cong 1.2705$	Where, $ 1.2705 - 1.2552 = 0.0153$
$y_2(0.91) \cong 1.4571$	Where, $ 1.4571 - 1.4434 = 0.0137$

In this case, it is possible to observe that:

$y_2(-0.745) = 1.3064$	Where, $ 1.3064 - 1.2906 = 0.0158$
$y_2(0) = 1$	Where, $ 1 - 1 = 0$

Since $y_3 = ax^2 + b$ and considering that points C' and D' lie on the graph of y_3 , we can determine coefficients a and b . In so doing, we have that:

i) $y_3(0) = 1 \Rightarrow b = 1$
ii) $y_3(0.7) = 1.2554 \Rightarrow 0.49a + 1 = 1.2554 \Rightarrow a = 0.5212$

Therefore, we have that:

$$y_3 = 0.5212x^2 + 1$$

From the previous equation, using points A', B' and E', we have that:

$y_3(-0.925) \cong 1.446$	Where, $ 1.446 - 1.4592 = 0.0132$
$y_3(-0.745) \cong 1.2893$	Where, $ 1.2893 - 1.2906 = 0.0013$
$y_3(0.91) \cong 1.4316$	Where, $ 1.4316 - 1.4434 = 0.0118$

In this case, it is possible observe that:

$y_3(0) = 1$	Where, $ 1 - 1 = 0$
$y_3(0.7) = 1.2554$	Where, $ 1.2554 - 1.2552 = 0.0002$

By using $y_4 = ax^2 + b$ and considering that points C' and E' lie on the graph of y_4 , we determine the coefficients a and b . In so doing, we have that:

i) $y_4(0) = 1 \Rightarrow b = 1$
ii) $y_4(0.91) = 1.4086 \Rightarrow 0.8281a + 1 = 1.4086 \Rightarrow a = 0.4934$

Therefore, we have that:

$$y_4 = 0.4934x^2 + 1$$

From the previous formula, using points A', B' and D', we have that:

$y_4(-0.925) \cong 1.4222$	Where, $ 1.4222 - 1.4592 = 0.037$
$y_4(-0.745) \cong 1.2738$	Where, $ 1.2738 - 1.2906 = 0.0168$
$y_4(0.7) \cong 1.2418$	Where, $ 1.2418 - 1.2552 = 0.0134$

In this case, it is possible to observe that:

$y_4(0) = 1$	Where, $ 1 - 1 = 0$
$y_4(0.91) = 1.4086$	Where, $ 1.4086 - 1.4434 = 0.0348$

Since $y_5 = ax^2 + b$ and considering that points A' and E' lie on the graph of y_5 we can determine the coefficients a and b . In so doing, we have that:

i) $y_5(-0.925) = 1.4699 \Rightarrow 0.855625a + b = 1.4699$ (I)
ii) $y_5(0.91) = 1.4086 \Rightarrow 0.8281a + b = 1.4086$ (II)

By subtracting equation II from equation I, we have that:

$$0.027525a = 0.0613 \Rightarrow a = 2.2271$$

By substituting the value of a in equation I or in equation II, we have that:

$$b = -0.4357$$

Therefore, we have that:

$$y_5 = 2.2271x^2 - 0.4357$$

In this case, it is possible to notice that this function does not clearly portray the question we were hoping to obtain. In this regard, we decided that it was not enough to observe that $y_5(0) = -0.4357$. On the other hand, since $y_6 = ax^2 + b$ and considering that points A' and D' lie on the graph of y_6 , it was possible to determine coefficients a and b . In so doing, we have that:

i) $y_6(-0.925) = 1.4699 \Rightarrow 0.855625a + b = 1.4699$ (I)
ii) $y_6(0.7) = 1.2554 \Rightarrow 0.49a + b = 1.2554$ (II)

In this case, we could determine the solution of the system formed for the equations I and II. Thus, we have that:

$$a = 0.5867$$

$$b = 0.9679$$

Therefore, we have that:

$$y_6 = 0.5867x^2 + 0.9679$$

From the previous equation and using points B', C' and E', we have that:

$y_6(-0.745) \cong 1.2935$	Where, $ 1.2935 - 1.2906 = 0.0029$
$y_6(0) = 0.9679$	Where, $ 0.9679 - 1 = 0.0321$
$y_6(0.91) \cong 1.4537$	Where, $ 1.4537 - 1.4434 = 0.0103$

It is also possible to observe that:

$y_6(-0.925) = 1.4699$	Where, $ 1.4699 - 1.4592 = 0.0107$
$y_6(0.7) = 1.2554$	Where, $ 1.2554 - 1.2552 = 0.0002$

In so doing, since $y_7 = ax^2 + b$ and considering that points B' and E' lie on the graph of y_7 , it is possible to determine the coefficients a and b . In this case, we have that:

i) $y_7(-0.745) = 1.3064 \Rightarrow 0.555025a + b = 1.3064$ (I)
ii) $y_7(0.91) = 1.4086 \Rightarrow 0.8281a + b = 1.4086$ (II)

The solution of the system formed by the equations I and II is given by

$a = 0.3743$
$b = 1.0986$

Therefore, we have that:

$$y_7 = 0.3743x^2 + 1.0986$$

From the previous equation and using points A', C' e D', we have that:

$y_7(-0.925) \cong 1.4189$	Where, $ 1.4189 - 1.4592 = 0.0403$
$y_7(0) = 1.0986$	Where, $ 1.0986 - 1 = 0.0986$
$y_7(0.7) \cong 1.282$	Where, $ 1.282 - 1.2552 = 0.0268$

In this case, it is possible to observe that:

$y_7(-0.745) = 1.3064$	Where, $ 1.3064 - 1.2906 = 0.0158$
$y_7(0.91) = 1.4086$	Where, $ 1.4086 - 1.4434 = 0.0348$

Table 2 shows a summary of each adjusted function.

Adjusted Function	Deviation shunting line in relation to the function hyperbolic cosine
Exponential	$0.0123 = \left(\frac{0.0107 + 0.0158 + 0 + 0.0002 + 0.0348}{5} \right)$
$y_1 = 0.5492x^2 + 1$	$0.01 = \left(\frac{0.0107 + 0.0142 + 0 + 0.0139 + 0.0114}{5} \right)$
$y_2 = 0.552x^2 + 1$	$0.0116 = \left(\frac{0.0131 + 0.0158 + 0 + 0.0153 + 0.0137}{5} \right)$
$y_3 = 0.5212x^2 + 1$	$0.0053 = \left(\frac{0.0132 + 0.0013 + 0 + 0.0002 + 0.0118}{5} \right)$
$y_4 = 0.4934x^2 + 1$	$0.0204 = \left(\frac{0.037 + 0.0168 + 0 + 0.0134 + 0.0348}{5} \right)$
$y_5 = 0.5867x^2 + 0.9679$	$0.0112 = \left(\frac{0.0107 + 0.0029 + 0.0321 + 0.0002 + 0.0103}{5} \right)$
$y_7 = 0.3743x^2 + 1.0986$	$0.0433 = \left(\frac{0.0403 + 0.0158 + 0.0986 + 0.0268 + 0.0348}{5} \right)$

Table 2: A Summary of Each adjusted Function

By checking the table above we can observe that the best fitting is given by a quadratic function, which has an equation of $y_3 = 0.5212x^2 + 1$. It is also possible to observe the worst fitting is given by the quadratic function $y_7 = 0.3743x^2 + 1.0986$. In order to visualize this assertion, we can analyze the graphs in figure 11.

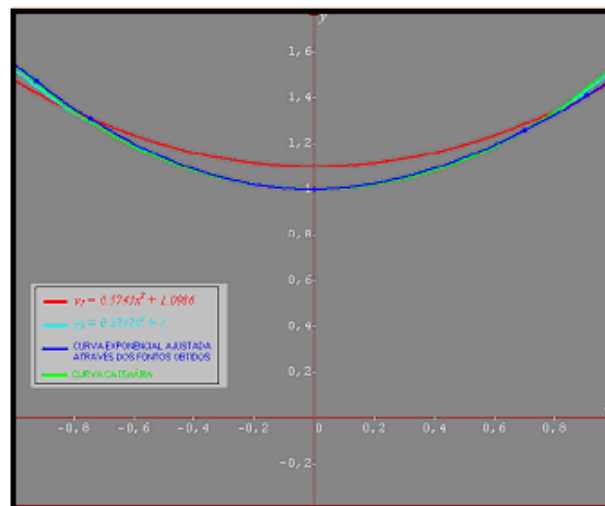


Figure 11: Graphs of three curves: exponential (blue), parabola (red), and catenary (green)

We can observe that the graph of the exponential function (blue) is similar to the graph of the catenary function (green). On the other hand, we also obtained a quadratic function (red) whose graph is also similar to the graph of the catenary function (green).

6.3 Some Considerations from the Mathematical Models

By observing the architectonic design on the wall of the Colégio Arquidiocesano, we were trying to relate them to several curves. Initially, we tried to check the similarity that seems to exist between these shapes and the exponential curve when we consider the shapes on the wall as a whole.

Further, by analyzing these shapes, individually, we were trying to relate them to exponential curves, parabolas, and catenaries.

However, when we visualized the shapes in only one part of the curve on the wall, we could observe the existence of similarities between the exponential curves, parabolas, and catenaries. In so doing, after examining the data collected when we measured various curves on the wall of the Colégio Arquidiocesano and trying to fit them to the exponential and quadratic functions through mathematical models we came to the conclusion that the curves on the wall of the Colégio Arquidiocesano closely approximate a *catenary curve*.

7 Final Considerations

Any study of mathematical modeling represents a powerful means for validating contextualized mathematical situations. This perspective forms the basis for significant contributions of a Freirean-based mathematical perspective in re-conceiving the discipline of mathematics in a pedagogical practice. The use of Freire's (1970) dialogical methodology is seen as essential in developing the curricular praxis of mathematical models by investigating the mathematics that is part of students' culture and constructing a curriculum that enable the enrichment of mathematical knowledge.

The use of ethnomodeling as pedagogical action for the teaching-learning of mathematics values the previous knowledge of the mathematical practices of the members of the community by developing students' capacity to assess the process of elaborating a mathematical model in its different applications and contexts by having started with the social context, reality and interests of the students. In the case of this article, the context in which mathematical models arose was the curves on the wall of the Colégio Arquidiocesano.

On the other hand, the discussions about the application of mathematical modeling into a cultural and common context allowed investigators to see that there are good physical and mathematical reasons why the curves used in architecture would be more like catenaries than parabolas. From the results of this investigation, we believe that mathematics that define the required shape of curves come from real physical laws, as the catenary does, while the parabola is much more of an idealized mathematical abstraction.

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