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Remote teaching of Differential Equations for engineering: modeling the spread of an epidemic

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ABSTRACT

OBJECTIVE: To investigate the possible contributions of mathematical modeling remote activities—due to the restrictions imposed by the COVID-19 pandemic—to learn differential equations. METHODS: The qualitative research was conducted with 117 students from 9 engineering programs at a federal university located in the state of Minas Gerais, Brazil, enrolled in the class Differential Equations I, in the first term of 2020. As for research methodology, mathematical modeling activities were planned from themes involving first and second order ordinary differential equations. These were developed and recorded on Google Meet and were subsequently evaluated by the participating students through a questionnaire. RESULTS: Results allowed us to indicate that the activities of the remote class shaped rich opportunities for students' motivation, allowed a unique exploration regarding the application of mathematical content related to first and second order ordinary differential equations, especially an activity which demanded modeling the spread of an epidemic, and also fostered a critical, albeit incipient, interpretation of reality. In addition, the results section sheds light on the challenges presented to students both in the academic context, from the institutional imposition of remote education, and in the social context, from the conditions prescribed by the pandemic that revealed massive socioeconomic differences among students. CONCLUDING REMARKS: The conclusions point to the importance of reflecting on the possible implications of the (post)pandemic context for the paths of current research in mathematics education, especially in Higher Education.

KEYWORDS: Remote Teaching. Differential Equations. Mathematical Modeling. Mathematics Education in Higher Education.

INTRODUCTION

Research concerning teaching and learning processes in higher education has been characterized by the investigation of phenomena related to the development of advanced thinking, as well as the factors that pose difficulties to the construction of more advanced mathematical concepts, thus expanding research on learning theories and teaching approaches related to such construction, among other topics (IGLIORI, 2009).

Thus, the way learning is conceived in higher education differs from how it is conceived in basic education. According to Igliori (2009), such distinction can be seen in the way one deals with curricular themes and in the way students act. Mathematical concepts are often viewed as “teaching objects”, as opposed to “learning objects”, which need to be shared between teachers and students. This way, higher education students achieve a greater burden for the success (or failure) of their learning (ALMEIDA; IGLIORI, 2013).

As a result, several studies have turned their attention to the teaching of differential and integral calculus (MACHADO, 2008; LOPES; REIS, 2019; REIS; COMETTI; SANTOS, 2019). As claimed by Machado (2008), some reasons for the poor performance in calculus learning are of cognitive order, that is, students do not yet have a cognitive structure necessary to understand calculus concepts. In didactics, according to this perspective, the challenge is to find the most appropriate methodology. In the epistemological context, the obstacles are in the gaps prior to the teaching of calculus.

This article presents a piece of research remotely conducted with engineering students on the use of mathematical modeling in the teaching of differential equations (DE). DE are here conceived as a theme or content integral to differential and integral calculus, although it is known that, in some curricula, ordinary differential equations (ODE) compose the program of calculus classes (in general, calculus II, III or IV) and, in other cases, they are taken as specific classes (called ODE or, simply, DE).

RESEARCH CONTRIBUTIONS ON DE TEACHING

From a bibliographic survey, Oliveira & Igliori (2013) examined research in the area of mathematics education in higher education aiming to present both problems in learning DE and the differentiated teaching proposals pointed out to tackle such difficulties. In line with the information stated in the introduction, the researchers found that the teaching of DE highlights the analytical solutions and algebraic manipulations involved, discussing, therefore, the difficulties students showed in previous content concerning basic mathematics and concepts of differential and integral calculus.

Another aspect worth of consideration is the learning difficulties related to applications in contextualized problem situations. In order to mitigate the aforementioned difficulties, most of the studies analyzed by the authors proposed a qualitative approach to DE, in a contextualized way, through problem situations, which can hold significant contributions to learning when associated with the students´ future practice and can, thus, increase their motivation. Therefore, an ideal scenario suggested by Oliveira & Igliori (2013) is to provide a balanced approach between analytical, graphical and numerical treatment, with the use of computational resources that promote DE learning.

Some research has also focused on the teaching of DE for engineering programs by either using contextualized problem situations or the analysis of physical phenomena (DULLIUS, 2009; BUÉRI, 2019) or using mathematical modeling under different perspectives (FERREIRA, 2010; FECCHIO, 2011). Although such studies have been carried out under different theoretical frameworks and investigative focuses, they suggest that, in the teaching of DE, analytical methods of resolution prevail when compared to the exploration of graphic interpretations. They also highlight students´ reluctance to a more qualitative treatment of DE and the rare use of technological resources when studying DE concepts and properties.

Lopes (2020b) and Lopes & Reis (2022), on the other hand, underlined the contribution of teaching DE using the contextualization/modeling of problem situations and/or natural phenomena for students' formative education, the strengthening of their critical competence, in addition to their motivation for learning, through an epistemological alternative that provides a combination of knowledge, skills, and competencies linked to students' daily lives.

Based on the contributions made by research in mathematics education in higher education, we now outline our research, addressing the teaching of DE. To do so, we first contextualize it in its locus and then indicate its methodology.

INTRODUCING THE *LOCUS* AND THE RESEARCH METHODOLOGY

In mid-March 2020, due to the outbreak of the COVID-19 pandemic, a federal university in the state of Minas Gerais in which this research was conducted officially adopted remote teaching in what was called an Exceptional Work Regime (EWR). Classes were then taught through synchronous meetings held on Google Meet. Communication between teachers and students took place during the classes and through weekly forums available on the Moodle platform, a virtual teaching and learning environment that had been adopted by the institution for several years.

Before the EWR, the class Differential Equations I had a workload of 60 hours, divided in two face-to-face classes held twice a week, totalizing four classes of 55 minutes each. After the EWR, at the scheduled times, synchronous online meetings were held instead, now with 117 students of the nine engineering programs of the aforementioned university, enrolled in two classes under the responsibility of the first author of this article, in the first academic term of 2020. Despite being a useful tool, the frequency of students in these meetings varied between 50% and 75%, a slightly lower frequency than that of face-to-face classes. In conformity with the syllabus, mathematical modeling activities—which are presented below—were developed. To investigate their contributions to DE learning was, therefore, the main objective of the research outlined in this work.

The activities were elaborated considering the perspective of mathematical modeling brought by Bassanezi (2002) and conceived in the educational perspective by Biembengut (2016), which uses the term "modeling" to refer to modeling in education, guided by the teaching of curricular content based on a general theme and, simultaneously, by the orientation of students to research an aspect of their interest, related to the theme. According to Biembengut, "Modeling is a teaching method with research in school boundaries and spaces, in any subject and phase of schooling: from the initial years of elementary school to the final years of higher education"[1] (BIEMBENGUT, 2016, p. 177).

The researcher contends that, in Higher Education, one can use physical and/or symbolic modeling, depending on factors such as: the number of students in a classroom, the content of the class, and students' prior experience with modeling. This perspective on modeling, however, is but one among several others, all of which "have in common, among their objectives, the use of mathematics for the study of real problems or situations" (ARAÚJO, 2002, p. 31).

Biembengut (2016) describes three phases in mathematical modeling so that modeling is performed in classroom: 1) Perception and apprehension (understanding and grasping the subject); 2) Comprehension and explanation (understanding and explaining the problem situation); 3) Signification and expression (expressing meaning)[2]. Within the phase of signification and expression, the mathematical model can be emphasized, for it is, according to Bassanezi (2002, p.19), the expression of the observed phenomenon through the synthetic representation of the observed elements, using a symbolic language.

Burghes & Borrie (1981, p. 13), in turn, in their book *Modelling with Differential Equations*, state that a model serves to explain some observed data, make predictions, and make a decision. Accordingly, to translate a real-world problem into a mathematical problem, one must assume some simplifications. In addition, the important variables must be identified and the relationships between them from then on must be made explicit, because "The assumptions and relationships constitute the 'mathematical model', and generally lead to a mathematical problem of some sort, which is solved for the relevant variables using appropriate mathematical techniques. The solutions must now be interpreted back in terms of the real problem" (BURGHES; BORRIE, 1981, p. 14).

Thus, as it is usual in the mathematical modeling process, Burghes & Borrie (1981) stress the importance of not only paying attention to teaching resolution techniques for DE, but also to considering the interpretations of the results of a problem. If the interpretations and the connection with the reality are not made, besides ceasing to be a modeling activity, students will not have the perception that mathematics has an important role in solving problems. In addition, as argued in this article, the critical/social discussion of a model should be established in any conception of mathematical modeling. Due to its relevance, it was also considered in our activities, especially for being aimed at future professionals of the most diverse areas of engineering.

PRESENTING THE RESEARCH ACTIVITIES

In order to perform the mathematical modeling activities, groups of four to six students were formed in the two classes (C1 and C2), chosen by the students themselves. With 52 students, the C1 organized 9 groups; whereas the C2, with 65 students, organized 11 groups. Throughout the term, all groups were able to perform mathematical modeling activities, meaning that no group has dropped out.

The mathematical modeling activities were divided into two parts. Each part presented two activities to be developed by the students. Thus, each group answered one activity of the first part, concerning first order ODE, and another of the second part, concerning second order ODE. In other words, each group carried out two activities. As modeling aims at "the study of real problems or situations" (ARAÚJO, 2002, p. 31), the following themes were suggested:

1st part:

1A) Absorption of alcohol in the body and risk of accidents

1B) Modeling an appropriate diet

2nd part:

2A) Consumer buying behavior

2B) Modeling the spread of an epidemic

These themes, inspired by Burghes & Borrie (1981), were suggested because they could be of interest to most students and were appropriate to the mathematical content of the class. A bibliographic research on mathematics education investigation that uses modeling with differential equations showed that the topics chosen are generally the same and do not have an implication in everyday problems or in the professional future of students (LOPES, 2021). In fact, according to Bassanezi (2002, p. 45), “the formulation of new or interesting problems is not always a very simple activity for a mathematics teacher”. Thus, the themes were chosen not only to motivate students, but also to facilitate discussions after obtaining mathematical solutions.

To conduct the mathematical modeling activities, the eight steps described in Laudares *et al*. (2017, p. 98) for ODE applications in physical phenomena were adapted so as to become the following didactic steps:

Step 1: Mathematizing by a physical law;

Step 2: Solving the differential equation of the model;

Step 3: Applying initial or boundary conditions;

Step 4: Substituting the given constants;

Step 5: Making the calculations requested in the problem situations explicit;

Step 6: Representing the mathematical model of the phenomenon;

Step 7: Designing graphics for the model;

Step 8: Briefly describing the phenomenon;

Step 9: Analyzing the equation of the model;

Step 10: Performing a critical analysis of the model.

Due to the characteristics of some problem situations described in the mathematical modeling activities, some steps have been subdivided into sub-items in order for students to better understand and solve the activities. To stimulate a critical discussion of the model, Steps 9 and 10 were added to the original model. These were aimed at the students ' critical perception of the model, acknowledging that, according to Laudares et al. (2017, p. 98), “this structure can be considered a pattern to be followed, with changes occurring according to the type of problem to be solved”. In line with the authors, these steps are a practical way to apply Bassanezi's (2002) modeling ideas. In addition, we noticed that the phases by Biembengut (2016) are articulated to these steps. In fact, Step 1 is related to phase (1) Perception and apprehension (understanding and grasping); Steps 2 to 7 are related to phase (2) Comprehension and explanation (understanding and explaining); and Steps 8 to 10 are related to phase (3) Signification and expression (expressing meaning).

Each mathematical modeling activity would start from a problem question to encourage students. A short introductory text would follow to contextualize the problem, providing some data clues. The groups would then move on to solving the models. The last task required a presentation of each group to the other students.

We now proceed to detail and discuss some results of the mathematical modeling activity 2B. The other activities are fully described and analyzed in Lopes (2020a).

MODELING THE SPREAD OF AN EPIDEMIC

The mathematical modeling activity 2B—modeling the spread of an epidemic—was inspired by section 7.4 of the book by Burghes & Borrie (1981, p. 150) and the articles by Weiss (2013) and Catlett (2015). It aimed to discuss the question: "How to formulate a model that shows the spread of an epidemic and what is the importance of such models nowadays?”

Based on the steps described above, the modeling process is now presented. In Step 1, the groups had a productive discussion in view of the context of the COVID-19 pandemic. They identified that the main variables, in view of the time, were the contaminated or susceptible individuals (S), the infected individuals (I) and those who were recovered (R), because they either died or were immunized. This is known as SIR model.

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One of the groups reported that this model is "reasonable for calculating the spread of pandemics" (LOPES, 2020a, p. 122). Both in this and in the last two steps, some groups acknowledged that other variables could have been included, such as the flow of people entering and leaving a city every day. These changes, however, were recognized to generate a much more complex ODE system to be solved, that is, the students observed that the proximity to reality is "proportional to the complexity of the model" (BASSANEZI, 2002, p. 125).

In Step 2, the groups were faced with the following ODE system:

$$\frac{dS}{dt} = -\beta SI, \quad \frac{dI}{dt} = \beta SI - \mu I, \quad \frac{dR}{dt} = \mu I$$

The total population is a constant indicated by N, so that $N=S(t)+I(t)+R(t)$. In this way, the sum of the three ODE of the system is a constant equal to zero. The constant β is the infection rate of the disease and $1/\mu$ is the average period of infection.

Solving an ODE system can be an arduous and laborious task. In the case of the SIR system, the situation was no different. Students reported several difficulties in solving the system due to the non-possibility of simplification to obtain more direct solutions. Thus, the groups were advised to obtain the functions involved (S*(t), I(t)* and *R(t)*) in an indirect way. For example, the function S(t) and the constant representing the initial number of susceptible individuals ($S_0 = S(0)$) can be obtained by means of the quotient between the equations of the system. Figure 1 brings the solution reached by a group of students and serves to illustrate the suggestion made.

Figure 1 — Obtaining the evolution of the number of susceptible individuals by one group of students

$$R(t) = 1 - S_0 * e^{-R_0(R(t)-R(0))}$$

Pois:

$$\frac{dS}{dR} = \frac{-\beta S}{\gamma}$$

$$\frac{dS}{dR} = R_0 * S$$

$$\int \frac{dS}{S} = \int R_0 dR$$

$$\ln(S) = R_0 * R(t) + K_1$$

$$e^{\ln(S)} = e^{R_0 * R(t) + K_1}$$

$$S(t) = e^{R_0 * R(t)} * e^{K_1}$$

$$e^{K_1} = K$$

$$S(t) = e^{R_0 * R(t)} * K$$

$$S(0) = S_0$$

$$S_0 = e^{R_0 R(0)} * K$$

$$K = \frac{S_0}{e^{R_0 R(0)}} = S_0 * e^{-R_0 R(0)}$$

$$S(t) = e^{R_0 R(t)} * S_0 * e^{-R_0 R(0)}$$

$$S(t) = S_0 * e^{-R_0(R(t)-R(0))}$$

$$Se\ S(t) + I(t) + R(t) = N$$

$$N = 1\ e\ I(t) = 0$$

$$R(t) = 1 - S(t)$$

$$R(t) = 1 - S_0 * e^{-R_0(R(t)-R(0))}$$

Source: Lopes (2020a).

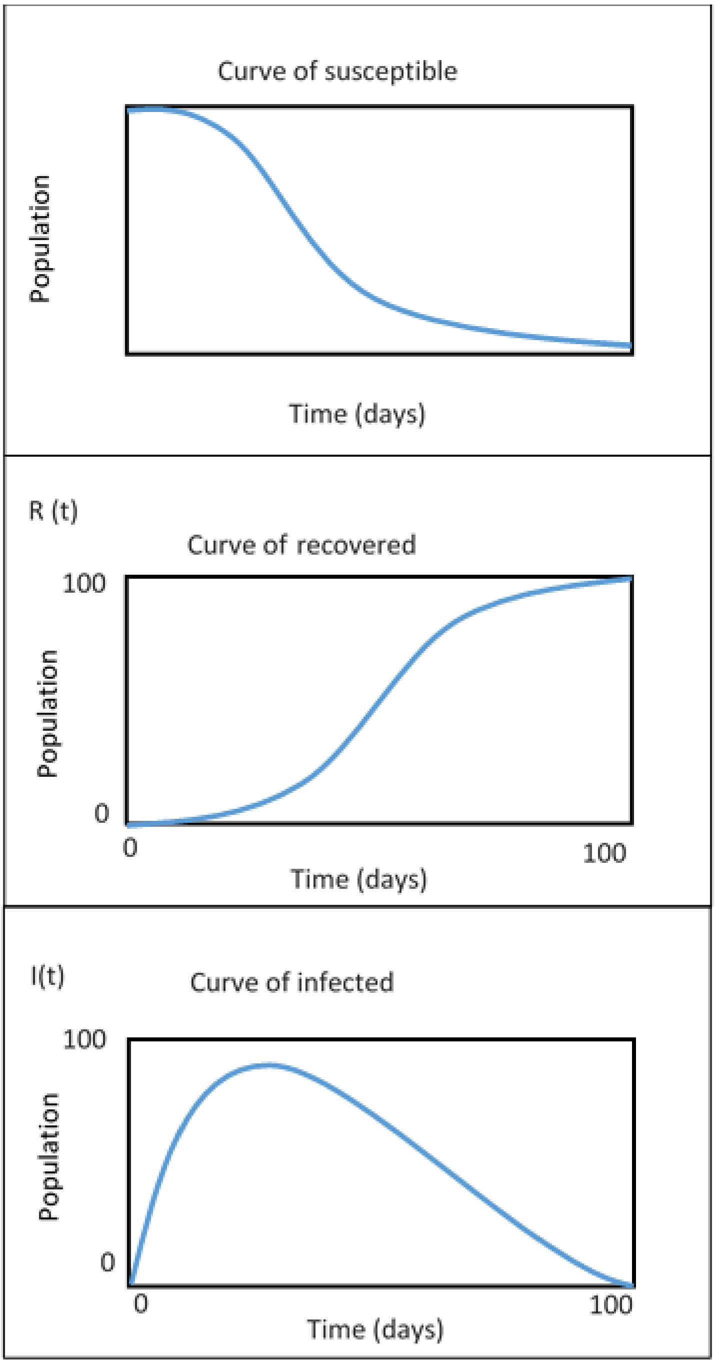
For Steps 3 to 6, the groups chose Brazilian cities to determine the constants of the system and the initial conditions. Cities such as Belo Horizonte, Brasilia, São Paulo, Rio de Janeiro, to name some, were among the cities chosen by the students to compare the results obtained with models released by the media and government agencies.

In Step 7, diverse groups reported considerable setbacks when designing the graphics. One of the reasons reported was the mathematical difficulty in both solving the SIR system and obtaining the functions of the system in indirect ways. In addition, some of the groups who presented lower difficulty in solving the mathematical portion of the ODE also reported difficulties in designing graphics. For this step, the use of technological resources and softwares, such as GeoGebra, was suggested. Despite the explanations and interventions to hep students in this step, some reported difficulties with "the tool used to plot graphics".

Besides the graphics directly related to SIR system functions, other graphics were suggested. As a particular case, the groups were encouraged to consider the vaccinated individuals. Until June 2020, this was a distant reality, since no laboratory had completely and successfully produced a vaccine against COVID-19. Given that a few modifications to the equations of the system were needed, the students found it difficult to solve the mathematical part of the new ODE obtained for the vaccinated case. As no group was able to solve it, no graphic was designed for this case.

Figure 2 shows the graphics produced by a different group of students, who analyzed the spread of COVID-19 in the city of Ipatinga, Minas Gerais. They produced graphics showing the evolution of susceptible S(t), recovered R(t), and infected I(t) individuals.

Figure 2 — Graphics showing the evolution of COVID-19 in Ipatinga-MG



Source: Adapted from Lopes (2020a).

Steps 8 to 10 did not involve mathematical calculations, but rather an interpretation of what was obtained. Although some groups presented some difficulties in solving the calculations, all of them were able to discuss the results in the last three steps. Step 8 represented the beginning of the discussions carried out by the groups, based on the mathematical results obtained, especially through the graphics. Analyzing the model used, one group recognized that ”although simple, it is deeply effective in its use, for it models the evolution of a susceptible, infected, and recovered population" (LOPES, 2020a, p. 139). Another group reported that there are “several factors that affect the trajectory of an epidemic” (LOPES, 2020a, p. 136) which are not covered by the model, although it tackles several important aspects of the spread of an epidemic. Many groups recognized the need for a quarantine to prevent a rapid increase in the number of infected individuals, which could cause various problems for the population. In addition, one group noticed that “the addition of vaccines” can lead to the situation that “vaccinated individuals go directly to the recovered group” (LOPES, 2020a, p. 140).

In the expressions created by the students to verify if the model was adequate to represent reality (Step 9), most of the groups attested that, although simplifications were made, the model built was close to reality. This result corroborates Bassanezi (2002), when he mentions that a model is not a faithful reproduction of reality, since there are variables that are not included and unforeseen events can happen in real situations. In this sense, one group concluded that “what will cause great variability in the results are human factors, such as treatment capacity or the measures a person takes to prevent the disease from spreading, for example.” (LOPES, 2020a, p. 141). One group still noticed that:

> The model at hand has important variables to be analyzed. It is of utmost importance to know the uninfected portion of the population that is still susceptible to the disease, to know the number of people who have been infected with the disease, and to be aware of people who are no longer able to infect or spread the infection. These are crucial variables to be considered in this epidemiological model. However, they are not enough. The virus undergoes several mutations, so the same person who was once infected and acquired antibodies can be infected again. Then, the [SIR] model becomes inefficient. (LOPES, 2020a, p. 141)

This group, as shown, commented on the adequacy of the model to reality and provided a brief critical discussion.

After Step 9, the groups analyzed the relevance and importance of the model in today's society (Step 10). One group concluded that:

> We have built a model that can be compared to one of the most used nowadays, which has extensive application and visibility by health professionals, which helps us conclude that our model could be applied in the real world. (LOPES, 2020a, p. 144)

In the same direction, another group restated the importance of quarantine, affirming that:

> The importance of such models nowadays can serve both for a government to control the virus with protective measures for people, such as quarantine and the like, thus knowing when to stop the flow of the country and when to return, and they can help by alarming people of the danger that the virus can cause and make everyone take necessary sanitary measures to prevent the virus from spreading. (LOPES, 2020a, p. 145)

These and similar comments indicated that the students verified the relevance of the model built in the context of a pandemic. For them, a projection predicted by a model can serve as a basis for governmental actions to avoid abrupt increases in the number of infected individuals.

CONTRIBUTIONS TO DE LEARNING

As described in Lopes & Reis (2021) regarding the contributions of mathematical modeling activities to DE learning, these activities provided a greater interest in DE, as unanimously pointed out by the students. This result can be associated with the fact that students, in general, are not used to engaging with "different methodologies", especially in Higher Education. The motivation gained through the activities led them to do research and get more involved with their studies.

Another result worth mentioning refers to the diverse applications of mathematics acknowledged by the students, including some contributions of modeling, all made clear in the answers to the questionnaire applied at the end of the mathematical modeling activities.

As a methodological alternative, mathematical modeling was widely accepted by the students, who confirmed the importance of knowledge built through the activities developed. In general, they understood the contributions of the use of modeling in the sense of a "teaching practice based on the precepts of mathematical modeling in education, demonstrating the role of the teacher as a mediator and making the student more autonomous in relation to their learning " (SCHELLER; BONOTTO; BIEMBENGUT, 2015, p. 17), which allowed us to reconcile theory and practice, uniting the world of academic mathematics with everyday-life mathematics, also collaborating for its critical interpretation.

Another contribution to learning pointed out by some students was the possibility of observing the connection that mathematics and modeling can build with other areas of knowledge. For other groups of students, the use of modeling in the class contributed to help them better understand previous content, especially Calculus I. Beyond the learning of the content, some groups presented a solution strategy that demanded further research activities. In addition to these results, all groups used software to present the graphic part of the model. This has certainly contributed not only to an understanding of the model, but also to DE learning.

In terms of issues that influenced learning, this study highlights the study environment, since some students did not have adequate spaces to study at home. This way, learning difficulties met the difficulties caused by the EWR implemented through remote teaching, which involved the need for students to study alone, away from their classmates, and to adapt to the remote system.

CONCLUDING REMARKS

In the context of the 2020 pandemic, teachers were forced to quickly make decisions about how to encourage their students to continue learning far from their classrooms (BAKKER; WAGNER, 2020). According to the researchers, there is a fear that the adoption of digital technologies could serve as an instrument to maintain the pedagogy of ”knowledge transmission", or a "laissez-faire", which could lead students to an unguided discovery. As an interesting example, Bakker & Wagner (2020) cite a request made by a Calculus student: “Please, even when you use technological tools, please send us a copy of your handwritten notes. It will be like when you write on the blackboard."

In the present research, the face-to-face class was deemed fundamental to students. During the first classes in remote education, students would regret being away from their classmates and from the physical space of the university. It was common to hear them asking when face-to-face classes would resume. For some students, a remote class was not a proper “class”. In the first weeks, both students and teachers needed a period of adaptation to the new regime of learning. After such period, the students adapted as they could to the new circumstances. The subsequent complaints were about learning itself. It seemed that some students were not certain about what they would actually learn during that academic term, besides feeling overwhelmed by the demands of the set of classes taken.

The remote teaching, on the other hand, was interactive. There were discussion forums, constant meetings with students through the available platforms and the promotion of activities that encouraged them. All this led to general satisfaction with the DE class, as demonstrated by the mostly positive comments regarding remote teaching made at the end of the university term. This result confirms the possibility of a quality remote teaching.

As for contributions to remote teaching, just as in a face-to-face classes, students need clear guidance and close monitoring during mathematical modeling activities. In remote education, since face-to-face contact is not possible, it is important that other types of meetings are considered. If possible, synchronous meetings should be done through video conferences for better interaction with students. In addition, asynchronous activities, such as a forum for questions and discussions, are also important to their learning process. Also, teachers should schedule extra hours to provide students with some time and space to discuss their doubts. Something valuable learned during the remote teaching was the way in which videoconferences were to be made and the forums were to be conducted: students must feel respected and have their particularities taken into account (whether internet-related needs or others), feeling part of the teaching and learning process.

Regarding the teaching of DE, the use of mathematical modeling can be very rich in terms of learning. The results demonstrated that the students showed greater interest in the class after they got involved with any project. Accordingly, the themes worked with students in modeling activities must be linked to their daily lives and/or be related to their future area of professional activity. When building a model, students may show difficulties in designing graphics. However, such visual aspects of the model provide a good basis for the conclusions to be made by them. The deadlines should be somehow flexible, since the groups may encounter difficulties or unforeseen events with respect to the technological devices used.

In addition, throughout the modeling process, students need to be stimulated to develop a critical capacity for discussion, which may initially thematize the model under construction and the variables involved. The discussion of the model can and should be directed to social, environmental, and economic issues. That is, if the entire process for the construction of the model is accompanied by relevant critical reflections, the modeling activity can stimulate students to assume a critical posture in their professional lives and in their roles as citizens in our society.

Just as the world is adapting to the new situations derived from the pandemic, companies are starting to adopt the hybrid model of work, and some have considered the current moment as a great opportunity for a "real historical change", leading to the "reinvention of concepts" such as office, work-from-home jobs, among others. These changes are global and are being considered to remain after the pandemic, generating a new way of working in several companies.

It is certain, however, that the period of social isolation brought negative aspects (ENGELBRETCH *et al*., 2020). For example, as this research showed, the socioeconomic differences may have a major impact on students´ lives, implying difficulties in adapting to the remote teaching and learning system.

Thus, given the changes caused by the pandemic, it is expected that educational institutions rethink and project the future of education. In the post-pandemic context of mathematics education, will remote teaching or hybrid teaching prevail? Or will we simply return to face-to-face classes, with no changes whatsoever?

In the attempt to look at the future of mathematics education in higher education, in addition to mathematical modeling, this research envisioned the possibility of approaching remote teaching through the use of digital tools for collaborative online studies (ENGELBRETCH; LLINARES; BORBA, 2020). In view of the results and considerations presented, the adaptation of students and their families to the remote system should be the subject of new research in mathematics education in higher education.

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ENSINO REMOTO DE EQUAÇÕES DIFERENCIAIS PARA ENGENHARIA: MODELANDO A PROPAGAÇÃO DE UMA EPIDEMIA

RESUMO

OBJETIVO: Investigar as possíveis contribuições da realização de atividades de Modelagem Matemática de forma remota, devido às restrições impostas pela pandemia de COVID-19 para a aprendizagem de Equações Diferenciais. MÉTODOS: A pesquisa, de cunho qualitativo, foi realizada na universidade federal do estado de Minas Gerais com 117 alunos, dos 9 cursos de Engenharia, matriculados na disciplina Equações Diferenciais I, no 1º semestre letivo de 2020. Como metodologia de pesquisa, planejamos atividades de Modelagem Matemática a partir de temas envolvendo Equações Diferenciais Ordinárias de 1ª e 2ª ordem que foram desenvolvidas e gravadas no Google Meet e, posteriormente, avaliadas pelos alunos participantes por meio de um questionário. RESULTADOS: Os resultados possibilitam afirmar que as atividades da disciplina, realizadas de forma remota, configuraram ricas oportunidades de motivação aos alunos participantes, permitiram uma exploração diferenciada das aplicações dos conteúdos matemáticos relacionados às Equações Diferenciais Ordinárias de 1ª e 2ª ordem - principalmente a atividade de Modelagem da propagação de uma epidemia - e, também, colaboraram para uma interpretação crítica da realidade, ainda que de forma incipiente. Particularmente nos resultados, são tecidas algumas considerações sobre os desafios enfrentados pelos alunos tanto no contexto acadêmico, a partir da imposição institucional do ensino remoto, como no contexto social, a partir das condições impostas pela pandemia que revelaram as enormes diferenças socioeconômicas dos alunos. CONSIDERAÇÕES FINAIS: As considerações finais do trabalho apontam para a importância de refletirmos sobre as possíveis implicações do contexto (pós)pandêmico para os caminhos da pesquisa vigente em Educação Matemática, especialmente no Ensino Superior.

PALAVRAS-CHAVE: Ensino Remoto. Equações Diferenciais. Modelagem Matemática. Educação Matemática no Ensino Superior.

NOTE

1. All quotes published in Portuguese were translated to English by the author.

2. The labels given to the phases were directly translated from Biembengut (2016), published in Portuguese. They are mentioned in Biembengut (2015), published in English, as they appear inside parentheses.

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